PCTS–Simons Collaboration workshop on Confinement and QCD Strings · 3 November 2026

Relativistic continuous matrix product states

$$|Q, R\rangle = \mathrm{tr}\left\{\mathcal{P}\exp\left[\int \mathrm{d}x\; Q\otimes\mathbb{1} + R\otimes a^{\dagger}(x)\right]\right\}|0\rangle$$

Antoine Tilloy

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Objective

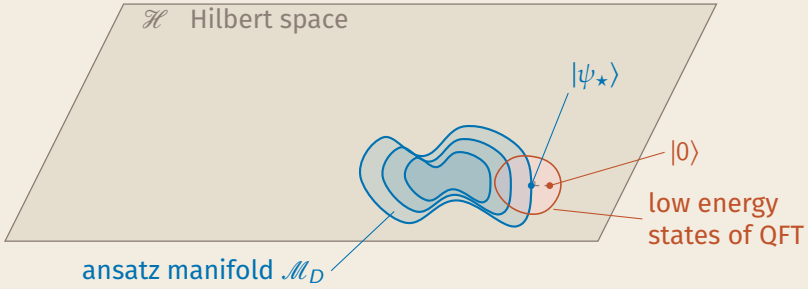
Solve generic QFT in $1+1$ dimensions,
to arbitrary precision and in the continuum

- Example: ϕ_2^4 model

$$H = \int_{\mathbb{R}} \mathrm{d}x \; \frac{:\pi^2:}{2} + \frac{:(\nabla\phi)^2:}{2} + \frac{m^2}{2} :\phi^2: + g :\phi^4:$$

and also coupled scalars, (massive) Sin(h)-Gordon, Schwinger, Ising field theory

Good candidate: variational method



$$|0\rangle \simeq |\psi_\star\rangle = \underset{|\psi\rangle \in \mathscr{M}_D}{\text{argmin}} \frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle}$$

What makes a good variational method

for more detail: Habilitation thesis, arXiv:2604.10284

Feynman's requirements

Difficulties in Applying the Variational Principle to Quantum Field Theories[1]

so I tried to do something along these lines with quantum chromodynamics. So I'm talking on the subject of the application of the variational principle to field theoretic problems, but in particular to quantum chromodynamics.

I'm going to give away what I want to say, which is that I didn't get anywhere! I got very discouraged and I think I can see why the variational principle is not very useful. So I want to take, for the sake of argument, a very strong view – which is stronger than I really believe – and argue that it is no damn good at all!

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1. **Extensive** parameterization ← IR
2. **Computable** expectation values
3. **Not oversensitive to short distances** ← UV

IR: many-body problem

discrete



exact diagonalization $\propto 2^n$

continuum



exact diagonalization $\propto e^{CL}$

IR: many-body problem

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n spins

exact diagonalization $\propto 2^n$

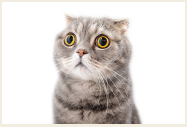
continuum



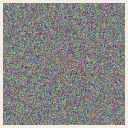
L

exact diagonalization $\propto e^{CL}$

Compression needed



cat image

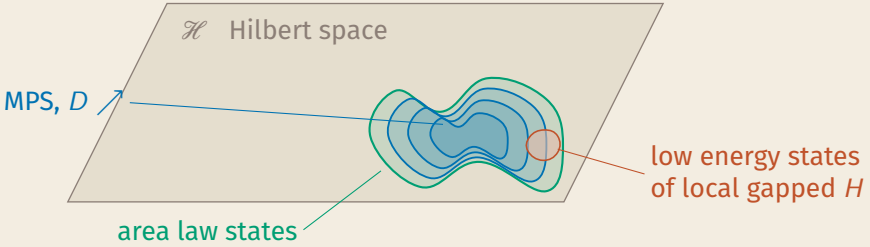


“typical” image

atypical $\Longrightarrow$ compressible

IR: matrix product states

Low energy states verify the area law



Matrix product states (MPS)

- cost: D^2 memory, D^3 compute
- error $1/\mathrm{superpoly}(D)$ for gapped ground states

$\longrightarrow$ numerically exactly solved

UV: short distance scale invariance

Entanglement entropy in QFT



$$S = -\mathrm{tr}\left[\rho_A \log \rho_A\right]$$

A relativistic QFT is critical at short distances:
divergent entanglement entropy

$$S \propto \log \Lambda \quad \Longrightarrow \quad D \propto \Lambda^{\kappa}$$

$\rightarrow$ naive continuum limit of an MPS fails

UV: Hamiltonian truncation

Choose the right factorization

Super-renormalizable theories are **free** (or simple CFT) in the UV:

$$H = H_0 + V \xrightarrow{\text{UV}} H_0 = \int \frac{\mathrm{d}k}{2\pi}\, \omega_k\, a_k^\dagger a_k$$

$\longrightarrow$ in the eigenmodes of H_0, short distances are disentangled

Relativistic continuous matrix product states

RCMPS in words

continuum limit of an MPS

Verstraete and Cirac 2010

+

Bogoliubov transform

RCMPS in equations

The ansatz:

$$|Q, R\rangle = \mathrm{tr}\left\{\mathcal{P}\exp\left[\int \mathrm{d}x\; Q \otimes \mathbb{1} + R \otimes a^\dagger(x)\right]\right\}|0\rangle$$

- Q, R: $D \times D$ matrices the variational parameters
- $|0\rangle$: Fock vacuum of H_0
- $a(x) = \frac{1}{2\pi}\int \mathrm{d}k\, e^{ikx} a_k, \quad [a(x), a^\dagger(y)] = \delta(x - y)$

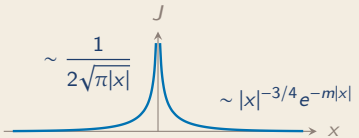
RCMPS in equations

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$$\phi(x) = \int \mathrm{d}y\; J(x-y)\left[a(y) + a^\dagger(y)\right]$$



Properties

$$|Q, R\rangle = \mathrm{tr}\left\{\mathcal{P}\exp\left[\int \mathrm{d}x\; Q \otimes \mathbb{1} + R \otimes a^{\dagger}(x)\right]\right\}|0\rangle$$

Feynman's checklist

1. extensive: $\mathcal{P}\exp\int$
2. computable: cost $O(D^3)$ $\longrightarrow$ non trivial
3. no UV problem: $|Q = 0, R = 0\rangle$ is the exact UV fixed point

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Optimization

- $e_0 = \langle Q, R|h|Q, R\rangle$ and ∇e_0 at cost $O(D^3)$ (backpropagation)
- Riemannian gradient descent

RCMPS: tradeoffs in the age of AI

For humans (since 2020):

- conceptually right

  Feynman requirements met

- technically heavy

  non-autonomous matrix ODEs, subtle convergence, complex optimization

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AI pays the technical cost → **the right tool is now cheap**

`github.com/AntoineTilloy/RcmpsCore.jl`

human CPU version (Julia) · AI GPU version (Python)

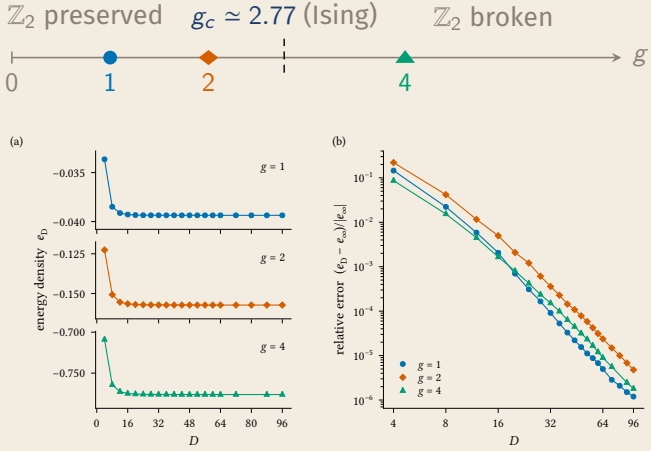
First example: ϕ_2^4

The model

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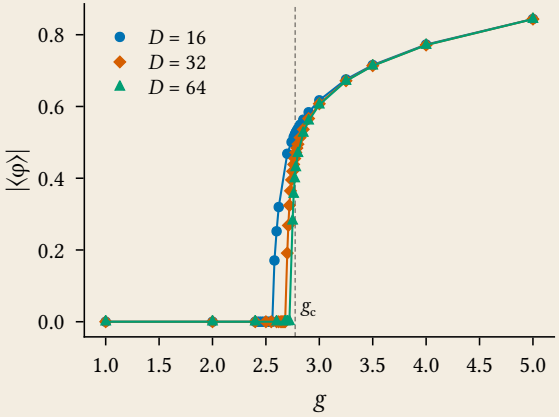
- normal ordering w.r.t. $m = 1$
- rigorously defined $\longrightarrow$ verifies Wightman axioms, etc.
- $\mathbb{Z}_2$ symmetry, broken for $g > g_c \simeq 2.77$

Energy density

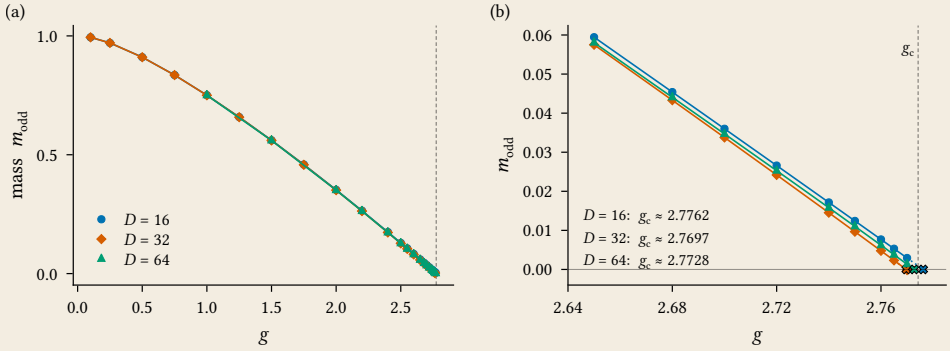


cost D^3, error plausibly $1/\text{superpoly}(D)$, close to machine precision

Magnetization



Mass



excitation ansatz: Pufu, Søgaard, Terlemes, arXiv:2610.13824

Schwinger model

work in progress with Karan Tiwana and Sophie Mutzel

The model

QED in $1+1$ dimensions, Coulomb gauge

$$H = \int \mathrm{d}x\, \bar{\psi}\left(-i\gamma^1\partial_x + m\right)\psi \;-\; \frac{g^2}{4}\int \mathrm{d}x\,\mathrm{d}y\; j^0(x)\,|x-y|\,j^0(y) \;+\; \theta\text{-term}$$

- no bosonization
- strongly relevant interaction
- confinement, θ-vacua

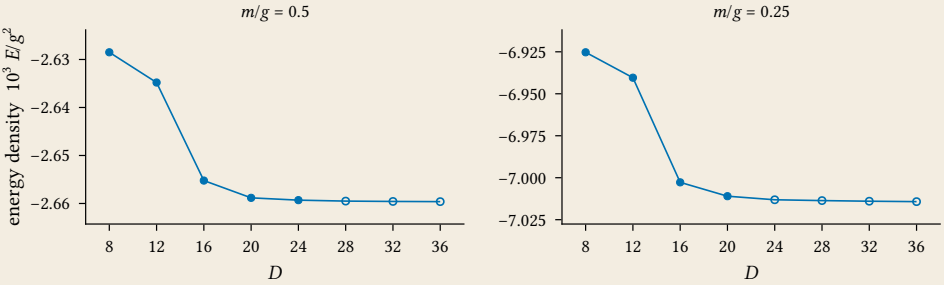
Ansatz

Fermionic RCMPS:

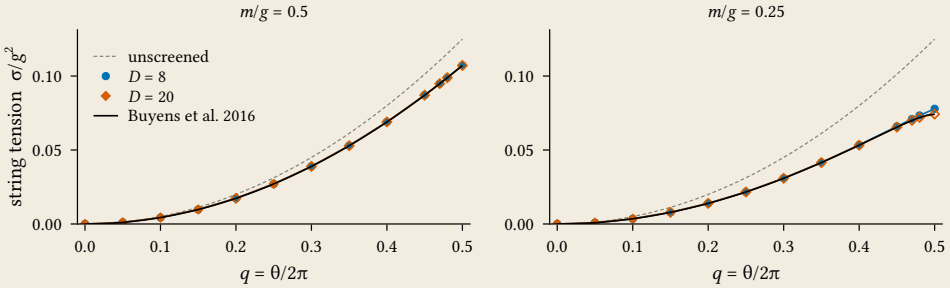
$$|Q, R_1, R_2\rangle = \mathrm{tr}\left\{\mathcal{P}\exp\left[\int \mathrm{d}x\; Q + R_1\, b_1^\dagger(x) + R_2\, b_2^\dagger(x)\right]\right\}|0\rangle$$

- $b_1(x), b_2(x)$: Fourier transforms of the fermionic modes
- **UV regularity**: $\{R_1, R_2\} = R_1^2 = R_2^2 = 0$
- **IR constraint**: global $U(1)$ constrains R_1 and R_2 further

Energy density

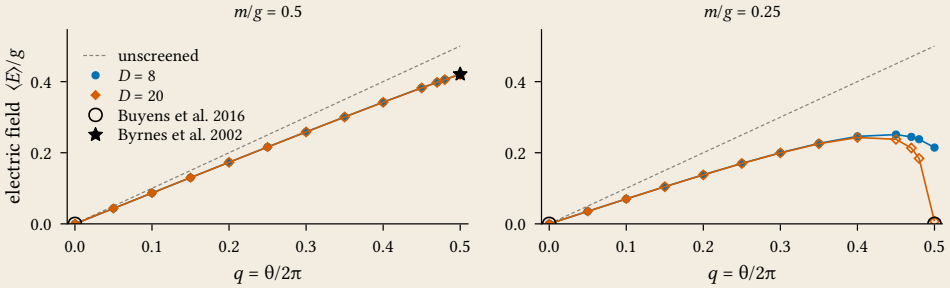


String tension



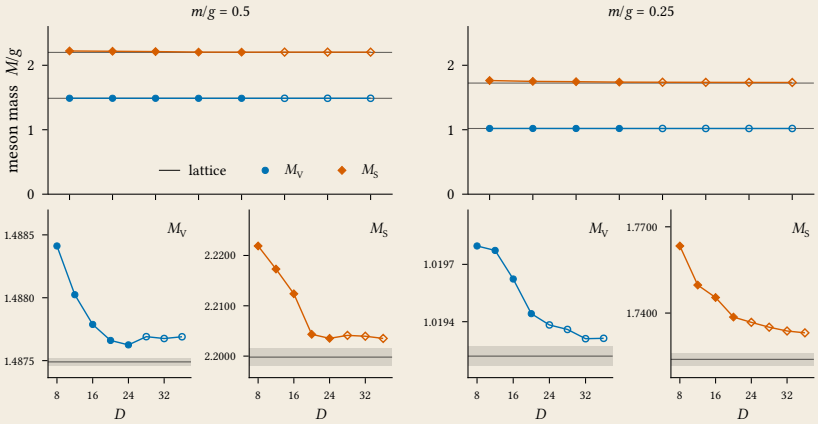
lattice comparison: Buyens et al., PRX 6, 041040 (2016)

Electric field



lattice comparisons: Buyens et al., PRX 6, 041040 (2016) · Byrnes et al., PRD 66, 013002 (2002)

Excitation masses



lattice comparison: Bañuls, Cichy, Jansen, Cirac, JHEP 11 (2013) 158

Conclusion

Summary

RCMPS: the right variational ansatz for $1+1$d QFT

- verify Feynman's requirements
- cost D^3, error $1/\text{superpoly}(D)$
- non-perturbatively accurate for ϕ_2^4, Schwinger and others

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RCMPS: the right variational ansatz for $1+1$d QFT

- verify Feynman's requirements
- cost D^3, error $1/\text{superpoly}(D)$
- non-perturbatively accurate for ϕ_2^4, Schwinger and others

Open problems

- beyond strongly renormalizable: perturbations with $\Delta \geq d/2$
- general perturbations of CFT: $H_{\text{free}} + V \;\longrightarrow\; H_{\text{CFT}} + V$
- $2+1$d and $3+1$d