Principle of least decoherence for Newtonian semiclassical gravity

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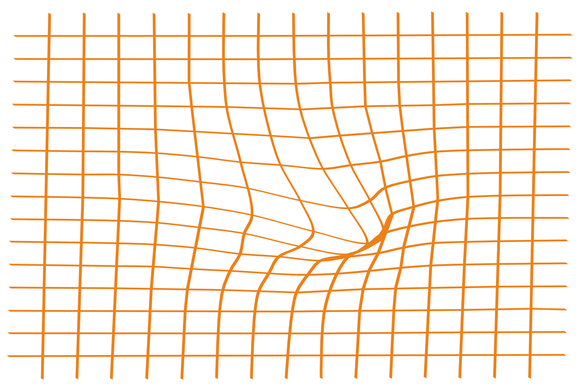


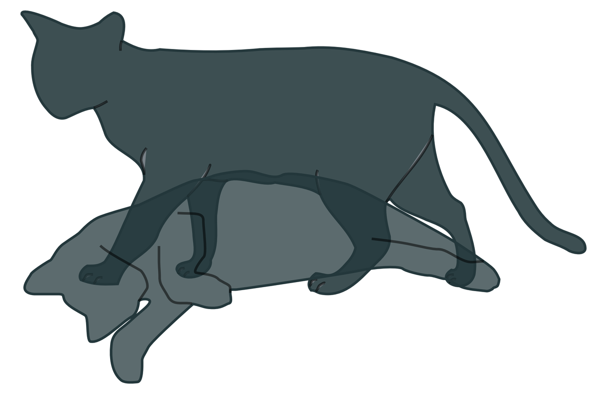
Classical space-time

Using the tools of quantum feedback, we construct a theory of semi-classical gravity, i.e. with:

classical space-time + **quantum matter**,

that avoids the inconsistencies of the standard semi-classical approaches in the non-relativistic limit. The model can be almost fully specified using a **principle of least decoherence**. We find that:

- The decoherence of the historical model of Diósi and Penrose is robustly recovered
- The $1/r$ law of gravity needs to be UV regularized for distances $\gg 10^{-15}\mathrm{m} \gg \ell_{\mathrm{Planck}}$



Quantum matter

Background

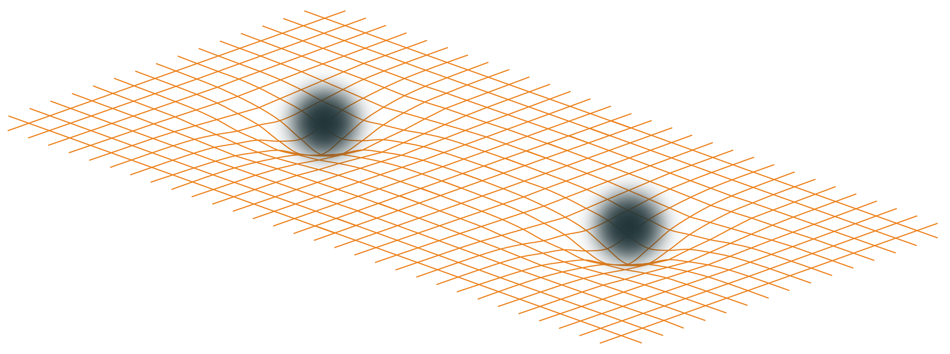
Fundamentally semiclassical gravity

1. A curved **space-time** modifies the dynamics of **quantum matter**:

$$\partial_\mu \to D_\mu$$

2. **Quantum matter** curves **space-time**:

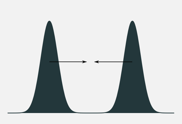
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \overset{?}{\propto} \underbrace{\hat{T}_{\mu\nu}}_{\text{operator}}$$



The 1st step is **known** and **well tested** physics (QFT in curverd space-time), the 2nd is **uncharted** territory. It is unclear how matter should source curvature. Two known options:

- Source with expectation values $\langle\cdot\rangle$
  $\implies$ Schrödinger-Newton (and variants)
- Source with a (formal) measurement signal

Reminder I: Schrödinger-Newton



The old **choice**, due to Møller and Rosenfeld, is to take $\langle\cdot\rangle$ to get operator$\to$ scalar:

$$\hat{T}_{\mu\nu}(x) \to \langle\Psi|\hat{T}_{\mu\nu}(x)|\Psi\rangle$$

In the non-relativistic lim. and for 1 particle, this means the grav. field φ is sourced by ψ^2:

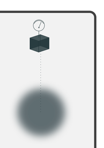
$$\nabla^2\varphi = 4\pi G\, m\, |\psi|^2$$

which gives the celebrated Schrödinger-Newton eq.:

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi - G\, m^2 \int d^3y\, \frac{\psi(y)^2}{|x-y|}\,\psi$$

which is manifestly **non-linear**
$\implies$ ~~Born rule~~, faster than light signalling

Reminder II: Continuous measurements



The continuous measurement of an operator $\mathcal{O}$ at a rate λ yields a measurement signal S_t:

$$S_t = \langle\mathcal{O}\rangle_t + \frac{1}{2\sqrt{\lambda}}\eta_t$$

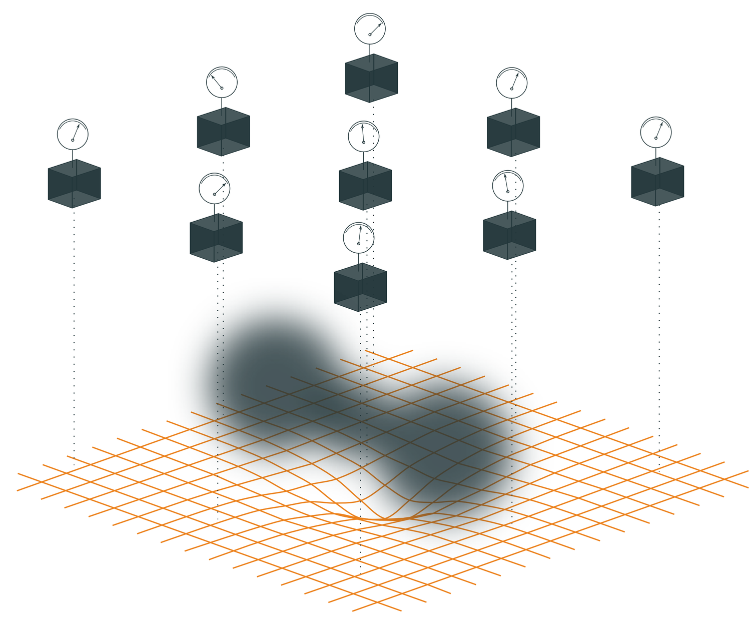
with η white noise. The corresponding back-action (collapse) on the state reads:

$$\partial_t|\psi_t\rangle = \Big[-iH + \sqrt{\lambda}\,(\mathcal{O} - \langle\mathcal{O}\rangle_t)\,\eta_t - \frac{\sqrt{\lambda}}{2}(\mathcal{O} - \langle\mathcal{O}\rangle_t)^2\Big]|\psi_t\rangle \qquad (1)$$

Model

1 – Measurement

We **imagine** that space-time is filled with detectors weakly measuring the mass density:



The equation for matter is now (1) with substitutions:
$\mathcal{O} \to \hat{\mathcal{M}}(x),\ \ \forall x \in \mathbb{R}^3$
$\lambda \to \gamma(x,y)$ coding detector strength and correlation
and there is a "mass density signal" $S(x)$ in every point.

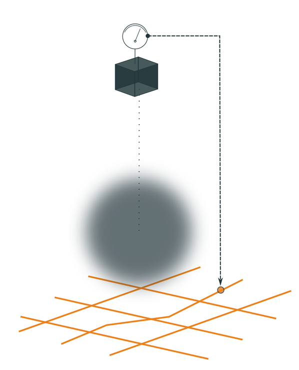
↓

2 – Feedback

We take the mass density signal $S(x)$ to source the gravitational field φ:

$$\nabla^2\varphi(x) = 4\pi G\, S(x)$$

which is **formally** equivalent to quantum feedback.



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3 – Result

Standard quantum feedback like computations give for $\rho_t = \mathbb{E}[|\psi_t\rangle\langle\psi_t|]$:

$$\partial_t\rho = -i\Big[H_0 + \frac{1}{2}\iint dx dy\, \mathcal{V}(x,y)\hat{\mathcal{M}}(x)\hat{\mathcal{M}}(y), \rho_t\Big] - \frac{1}{8}\iint dx dy\, \mathcal{D}(x,y)\Big[\hat{\mathcal{M}}(x), \big[\hat{\mathcal{M}}(y), \rho_t\big]\Big],$$

with the gravitational pair-potential

$$\mathcal{V} = \left[\frac{4\pi G}{\nabla^2}\right](x,y) = -\frac{G}{|x-y|},$$

and the positional decoherence

$$\mathcal{D}(x,y) = \left[\frac{\gamma}{4} + \mathcal{V}\circ\gamma^{-1}\circ\mathcal{V}^{\top}\right](x,y)$$

Hence the expected pair potential has been generated consistently at the price of more decoherence.

Nice features of the approach
1. Macroscopic superpositions collapse
2. The Born rule holds / no FTL signalling

Principle of least decoherence

Basics

There is still a (functional) degree of freedom $\gamma(x,y)$:

- Large $\|\gamma\|$ $\implies$ strong "measurement" induced decoherence
- Small $\|\gamma\|$ $\implies$ strong "feedback" decoherence

There is an optimal kernel that minimizes decoherence. Diagonalizing in Fourier, one gets a global minimum for

$$\gamma = 2\sqrt{\mathcal{V}\circ\mathcal{V}^{\top}} = -2\mathcal{V}$$

Hence:

$$\mathcal{D}(x,y) = -\mathcal{V}(x,y) = \frac{G}{|x-y|}$$

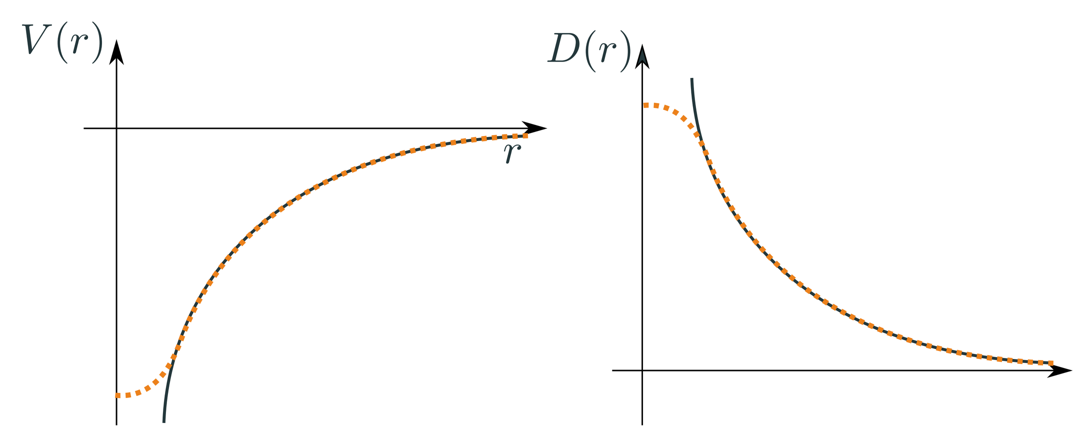
This is just the decoherence kernel of the Diósi-Penrose model (erstwhile heuristically derived)!

Regularization

Even for the minimal decoherence prescription, the decoherence is **infinite**. Adding a regulator at a length scale σ has 2 effects:

- It tames decoherence, making it finite
- It regularizes the pair potential $\propto \frac{1}{r}$ for $r \lesssim \sigma$

$\implies$ there is a **trade-off**.



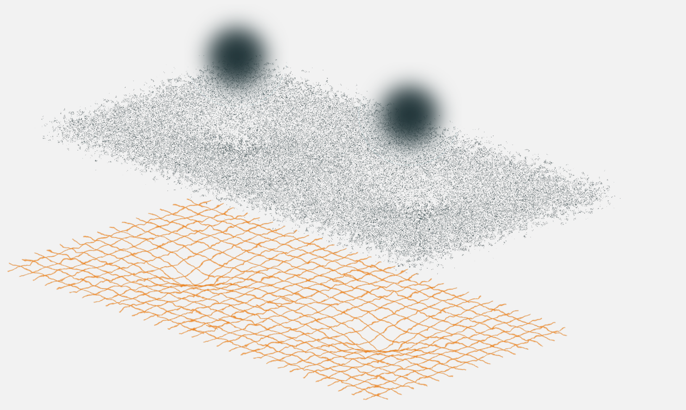
Experimentally:

$$\underbrace{10^{-15}\mathrm{m}}_{\text{decoherence constraint}} \le \sigma \le \underbrace{10^{-4}\mathrm{m}}_{\text{gravitational constraint}}$$

Importantly $\sigma > \ell_{\mathrm{Compton}} \gg \ell_{\mathrm{Planck}}$. Semiclassical gravity (if it takes this form) is falsifiable at atomic/molecular wavelengths.

Status of the signal

The signal is the continuous equivalent of the flashes in collapse models. It can be stripped of its operationalist interpretation and taken as primitive ontology, the bridge between wave functions and space-time.



Take home message

Semiclassical gravity in the Newtonian limit is consistent and *almost* parameter free. The price to pay is:

- A short distance regularization of the $1/r$ Newtonian potential
- Gravitational decoherence (inversely proportional to the cutoff)

Because of this trade-off, the theory is falsifiable for all regulators and soon experimentally testable.

Relevant references

[1] "*Sourcing semiclassical gravity from spontaneously localised quantum matter*" A. Tilloy, L. Diósi, *Phys. Rev. D* **93** (2016)

[2] "*A classical channel model for gravitational decoherence*", D Kafri, J M Taylor and G J Milburn, *New J. Phys.* **16** (2014)

[3] "*Probing Gravitational Cat States in Canonical Quantum Theory vs Objective Collapse Theories*", M. Derakhshani, arXiv:1609.01711

[4] "*Principle of least decoherence in semiclassical gravity*", A. Tilloy, L. Diósi, arXiv:1706.01856