Controlling Quantum Flux Through Measurement

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Introduction

Classically, no transfer occurs between two equally filled reservoirs no matter how one looks at them, but the situation can be different quantum mechanically. This paradoxically surprising phenomenon rests on the distinctive property of the quantum world that one cannot stare at a system without disturbing it. It was recently discovered that this seemingly annoying feature could be harnessed to control small quantum systems using weak measurements. Here we present one of the simplest models – an idealised double quantum dot – where by toying with the dot measurement strength, i.e. the intensity of the look, it is possible to create a particle flux in an otherwise completely symmetric system. The basic property underlying this phenomena is that measurement disturbances are very different on a system evolving unitarily and a system evolving dissipatively.

Main equations and model

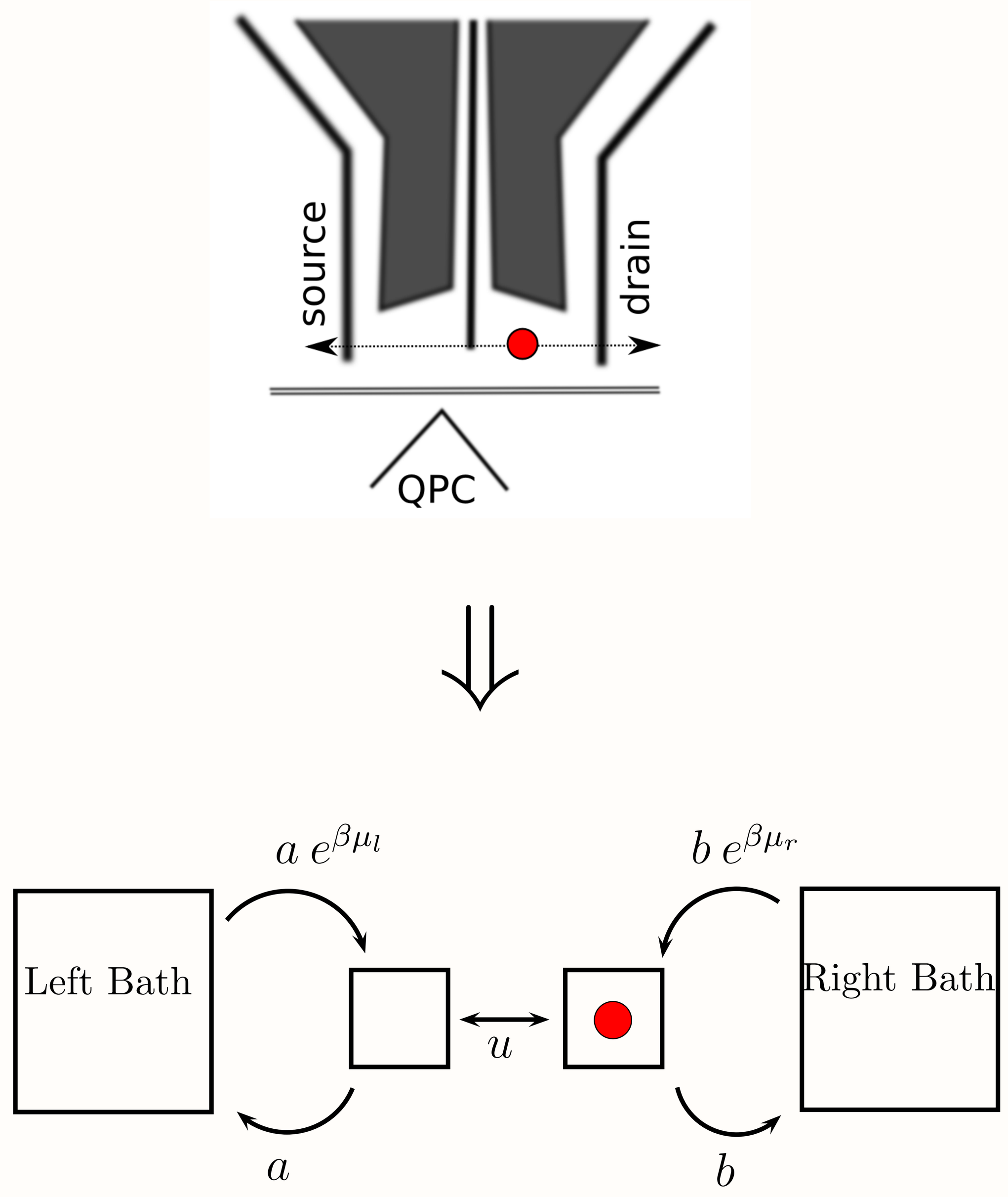

Figure: Schematics of a double quantum dot (DQD) measured by a quantum point contact (QPC) and the idealised model we derive from it, with an electron on the right. The arrows represent the possible dynamical processes.

System Hilbert space (encoding eletron position)

$$\mathcal{H} = \text{Vect}\{|0\rangle, |L\rangle, |R\rangle\}$$

Full evolution of the system

$$d\rho_t = d\rho_t^{\text{tunnel}} + d\rho_t^{\text{bath}} + d\rho_t^{\text{measure}} \qquad (1)$$

Unitary tunnelling

$$d\rho_t^{\text{tunnel}} = -i[H, \rho_t]dt$$

$$H = u\,(|R\rangle\langle L| + |L\rangle\langle R|)$$

Dissipative coupling with the bath

$$d\rho_t^{\text{bath}} = \mathcal{L}^{\text{bath}}(\rho_t)\,dt$$

$$\mathcal{L}^{\text{bath}}(\rho_t) = (L_{\sigma_l^+} + L_{\sigma_l^-} + L_{\sigma_r^+} + L_{\sigma_r^-})(\rho_t)$$

Here $L_\sigma(\rho) = \sigma\rho\sigma^\dagger - \frac{1}{2}\{\sigma^\dagger\sigma, \rho\}$ with $\sigma_l^+ = \sqrt{a}e^{\beta\mu_l/2}|L\rangle\langle 0|$, $\sigma_l^- = \sqrt{a}|0\rangle\langle L|$, $\sigma_r^+ = \sqrt{b}e^{\beta\mu_r/2}|R\rangle\langle 0|$ and $\sigma_r^- = \sqrt{b}|0\rangle\langle R|$

Measurement back-action

$$d\rho_t^{\text{measure}} = L_{\mathcal{O}}(\rho_t)\,dt + D_{\mathcal{O}}(\rho_t)dW_t$$

Here $D_{\mathcal{O}}(\rho) = \{\mathcal{O}, \rho\} - 2\rho\,\text{tr}(\mathcal{O}\rho)$ with $\mathcal{O} = h_l|L\rangle\langle L| + h_r|R\rangle\langle R| + h_0|0\rangle\langle 0|$ and $h_0 = 0$, $h = h_l = -h_r$ to simplify.

Measurement results

$$dX_t = 2\,\text{tr}(\mathcal{O}\rho_t)dt + dW_t \qquad (2)$$

Parametrisation (stable form)

$$\rho = \begin{bmatrix} Q_0 & 0 & 0 \\ 0 & Q_l & iK/2 \\ 0 & -iK/2 & Q_r \end{bmatrix} \qquad (3)$$

Quantum jumps

In the strong measurement limit, the behaviour of ρ gets jumpy even if the driving measurement noise is Gaussian.

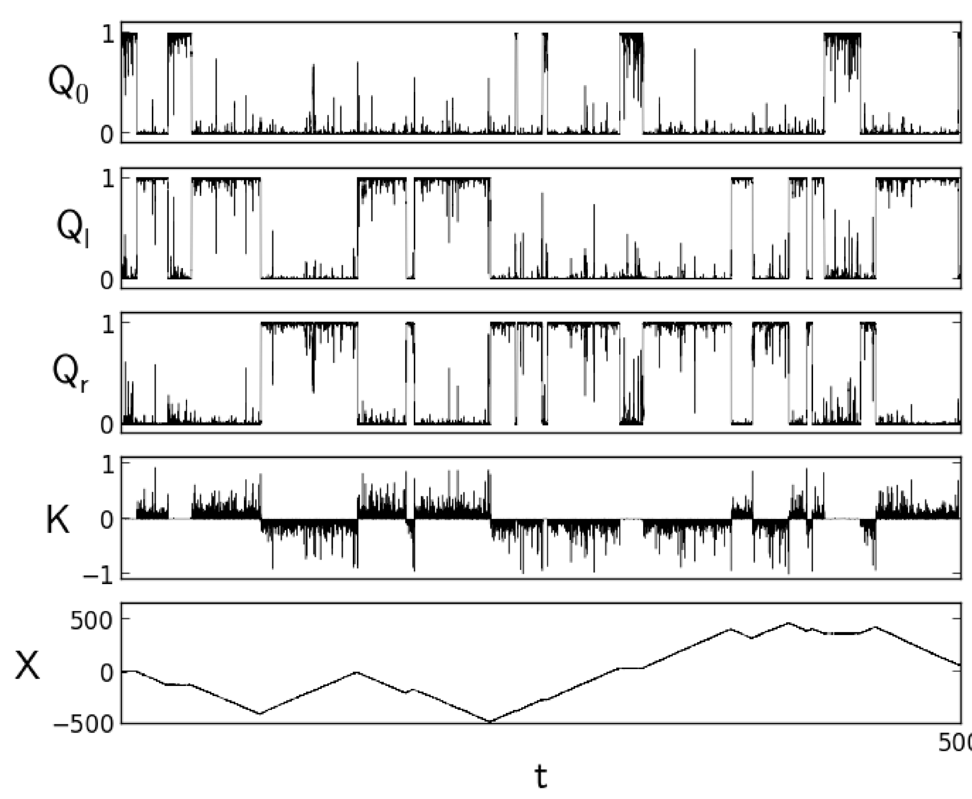

Figure: A trajectory with $a = b = 0.02$, $\beta\mu_l = -\beta\mu_r = 1.0$, $h_r = -h_l = 7.0$ (i.e. strong measurement) and $u = 1.0$.

Computing the jump rates

The jump rates can be computed from (1) for strong measurement with Kramers-like approximations. $\lambda_{0L}, \lambda_{0R}, \lambda_{L0}, \lambda_{R0}$ converge to constants, (no *Zeno effect*), for large measurement but $\lambda_{RL} = \lambda_{LR} = u^2/h^2 \rightarrow 0$ (*Zeno effect*)

Redefining the flux

Jumps give a new classically inspired way to compute the quantum flux. One simply has to count the number of transitions $L \rightarrow R$ minus $R \rightarrow L$.

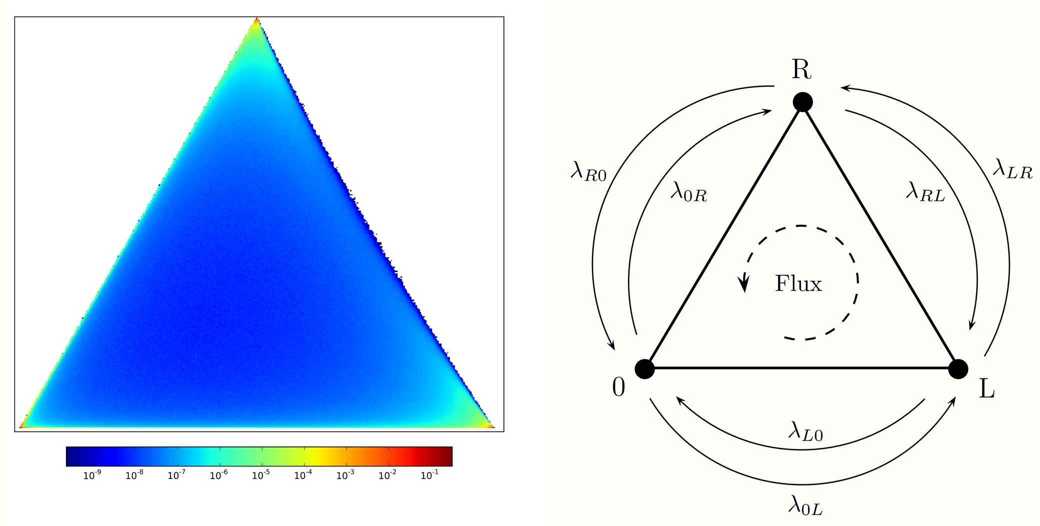

Figure: Invariant measure in the $Q_0 + Q_l + Q_r = 1$ domain computed with the Monte-Carlo method illustrating the approximation of the evolution as a Markov chain on a simplex.

This new definition for the flux can then be computed numerically or with a Markovian approximation and be compared with the standard quantum flux $\text{tr}(\hat{J}\rho_{stat})$. The different methods are consistent for strong measurements but only the classically inspired computation is possible in the presence of feedback.

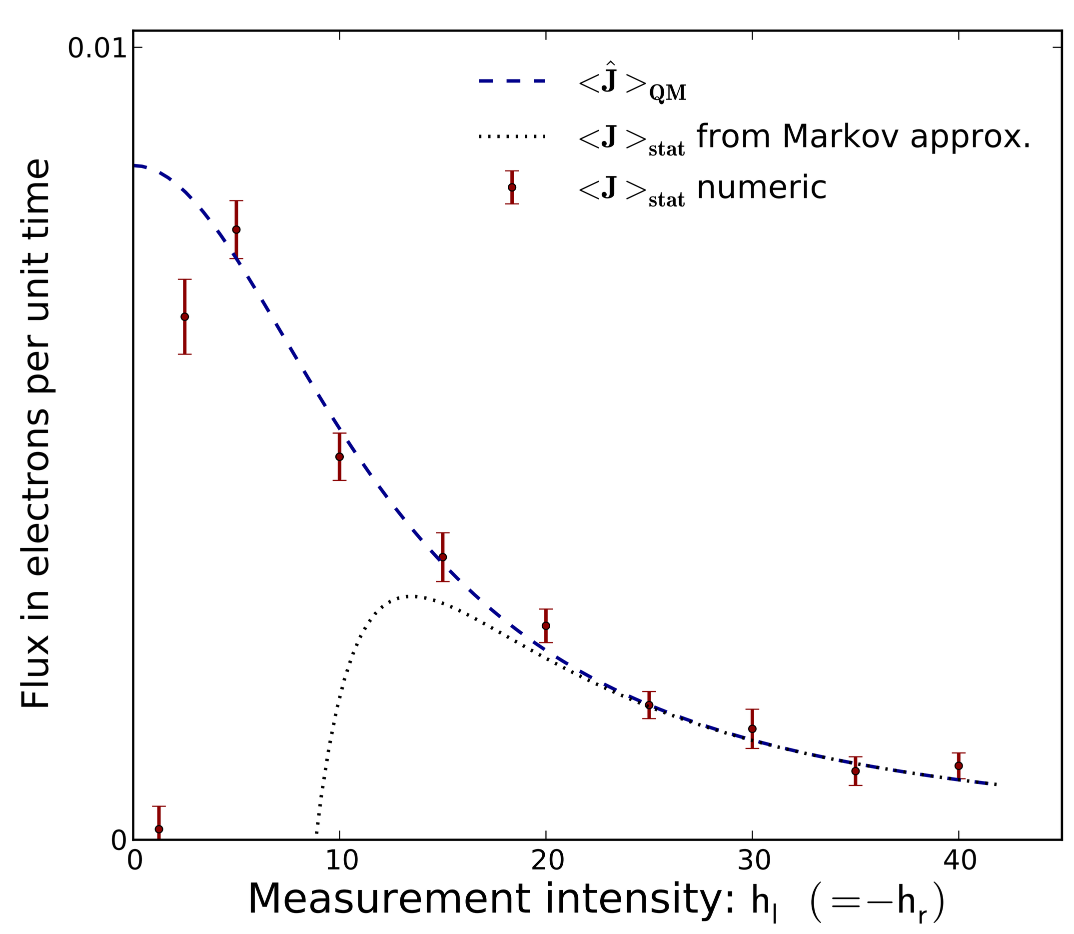

Figure: Comparison between purely analytic (quantum mechanical average) flux, Markov approximated and numeric statistical flux as a function of the measurement strength with $a = b = 0.02$ and $u = 1.0$. Constant difference of potential $\beta\mu_l = -\beta\mu_r = 2.0$ and the measurement strength varies. The three methods converge for strong measurements.

Feedback

One can use the fact that the transition rates do not have the same dependence in the measurement strength (dissipative junctions cannot be Zeno frozen) to bias the flux with a controlled stroboscopic measurement.

Idea

Measure strongly ($h = h_{max}$) when the electron is in the right dot and mildly otherwise ($h = h_{min}$) to create a non-zero additional average flux from left to right.

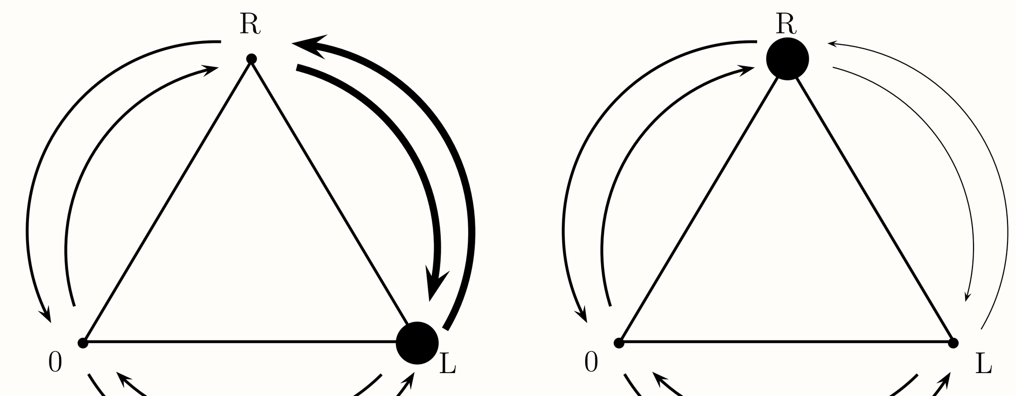

Figure: Influence of the feedback scheme on the transition rates.

Results

The average flux with feedback can be estimated with the Markovian approximation or numerically computed directly from equation (1) with a time dependant h. The Markov approximation gives:

$$\langle J\rangle_{\text{stat}} \propto u^2 \left(\frac{e^{\beta\mu_l}}{h_{min}^2} - \frac{e^{\beta\mu_r}}{h_{max}^2}\right) \qquad (4)$$

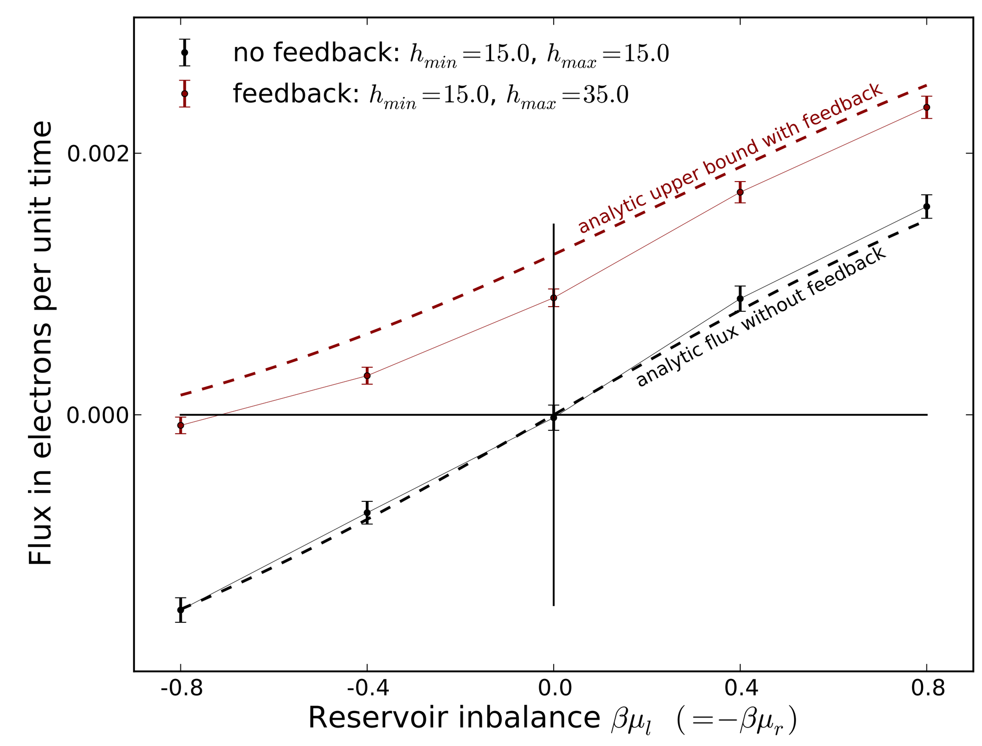

Figure: Average electron flux as a function of the difference of chemical potential between the two electron baths. The analytic expressions in the Markovian limit are shown in dashed lines. The feedback scheme indeed increases the flux in the DQD and is even able to counter a small difference of potential ! ($a = b = 0.02$, $u = 1.0$)

Conclusion

Using a simple feedback scheme, in the sense that it relies on the measurement strength of the apparatus only, it is possible to create, control or reverse a particle flux in a quantum system. The controlled stroboscopic measurement scheme we propose bears strong similarities with an effective Maxwell Daemon. This effect shows that adaptive measurements can have dramatic effects enabling transport control but possibly inducing biases in the measurement of macroscopic quantities if not handled with care.

References

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