An alternative to the Schrödinger-Newton approach to non-relativistic gravity in the quantum regime

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Main objective:

We construct a theory of semi-classical gravity, i.e. with:

quantum matter + classical space-time,

that avoids the inconsistencies of the standard Schrödinger-Newton approach in the non-relativistic limit.

Introduction

Classical gravity in a nutshell

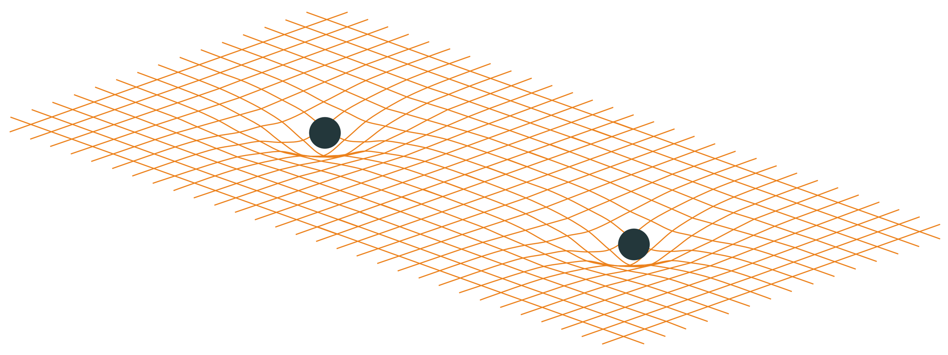
Classical gravity has two distinct facets:

1. A curved **space-time** modifies the dynamics of **matter**:

$$\partial_\mu \to D_\mu$$

2. **Matter** curves **space-time**:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \propto T_{\mu\nu}$$



Semi-classical gravity in a nutshell?

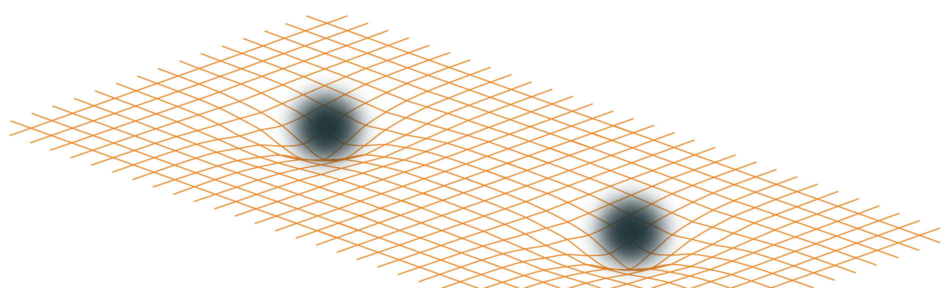
Semi-classical gravity should work in the same way:

1. A curved **space-time** modifies the dynamics of **quantum matter**:

$$\partial_\mu \to D_\mu$$

2. **Quantum matter** curves **space-time**:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \overset{??}{\propto} \underset{\text{operator}}{\hat{T}_{\mu\nu}}$$



The 1$^{\text{st}}$ step works, but the 2$^{\text{nd}}$ is problematic. It is unclear how matter sources curvature with "fuzzy" particles.

Standard approach The old **choice**, due to Møller and Rosenfeld, is to take $\langle\cdot\rangle$ to get operator→ scalar:

$$\hat{T}_{\mu\nu}(x) \to \langle\Psi|\hat{T}_{\mu\nu}(x)|\Psi\rangle$$

Schrödinger Newton In the non-relativistic lim. and for 1 particle, this means the grav. field φ is sourced by ψ^2:

$$\nabla^2\varphi(x) = 4\pi G m \psi^2(x)$$

which gives the celebrated Schrödinger-Newton equation:

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi - G m^2 \int d^3y \frac{\psi(y)^2}{|x-y|}\psi$$

which is manifestly **non-linear**.

Problems of the standard approach

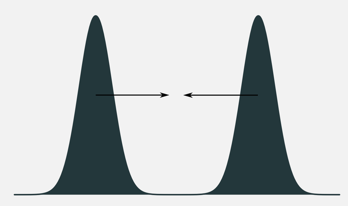
Because of the deterministic non-linearity, this canonical approach is ill-suited for a fundamental description of Nature:

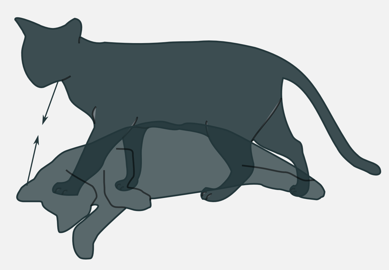
- The Born rule breaks down
- One can signal faster than light

1-particle self interaction

The Schrödinger-Newton equation implies that a single particle self-interacts.

The two blobs of the wave function attract each other.

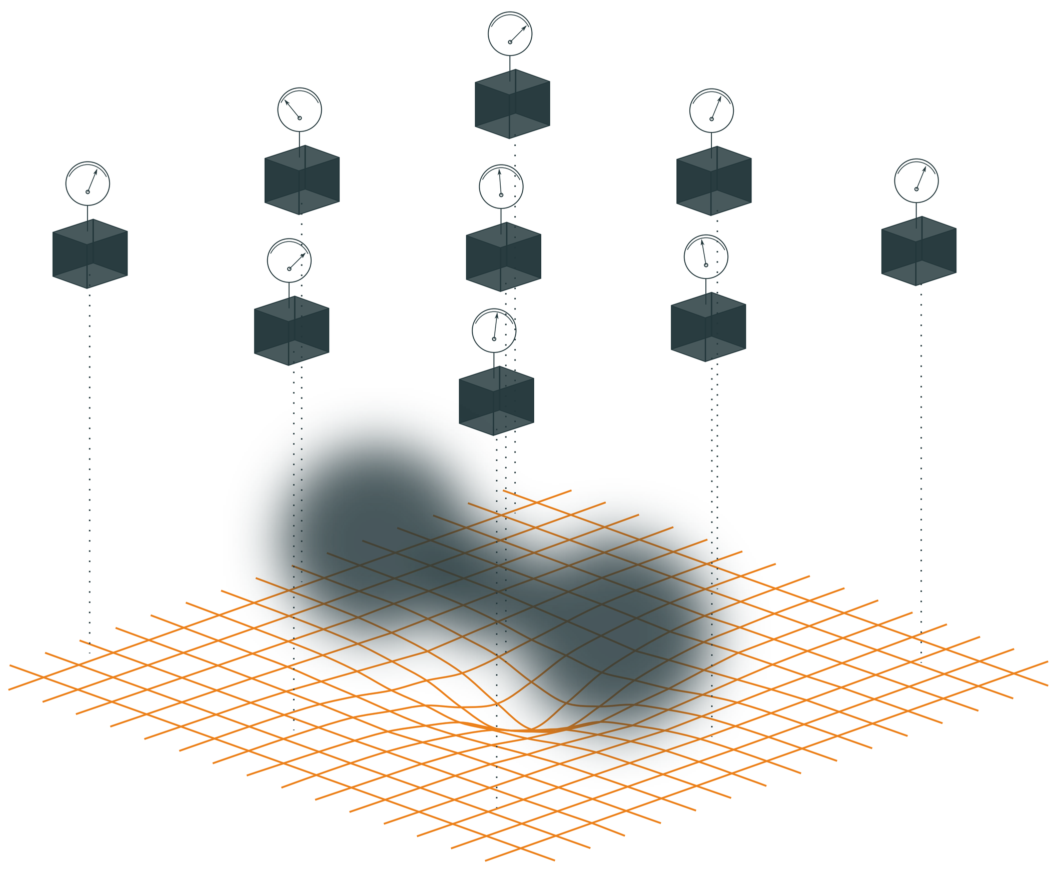




But worse, even **fully decohered** macroscopic superpositions attract each other!

This phenomenon is often assumed to be a generic property of semi-classical theories. Our construction shows that this is not the case.

Q: What can play the role of a mass density without breaking the linearity of the dynamics?
A: The **signal** from virtual detectors (weakly) measuring the mass density in every point! → feedback



The theory

To construct our theory, we assume the fundamental equations of Nature are **as if** a regularized mass density $M_\sigma(x)$ were continuously measured in every point x of space:

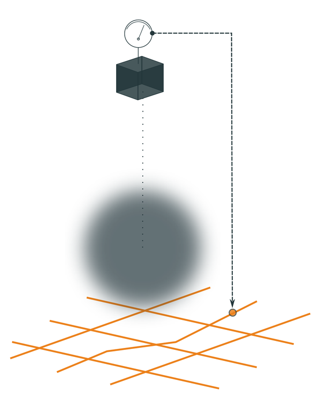
$$\frac{d\rho_t}{dt} = -i[H,\rho_t] + \int dx \gamma \mathcal{D}[M_\sigma(x)]\rho_t + \sqrt{\gamma}\mathcal{H}[M_\sigma(x)]\rho_t w_t^x$$

Sourcing the gravitational field

We now take the mass density signal $\mathcal{S}(x)$ to source the gravitational field φ:

$$\nabla^2\varphi(x) = 4\pi G m \mathcal{S}(x)$$

which is **formally** equivalent to quantum feedback.

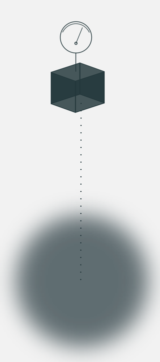


Continuous measurement theory

The measurement of some observable $\mathcal{O}$ –here typically the smeared mass density $\hat{M}_\sigma(x)$ in some point x– adds a stochastic term to the evolution.

$$\frac{d\rho_t}{dt} = -i[H,\rho_t] + \mathcal{D}[\mathcal{O}](\rho_t) + \mathcal{H}[\mathcal{O}](\rho_t)w_t$$

where w_t is white noise in Itô convention and:



- $\mathcal{D}[\mathcal{O}](\rho) = \mathcal{O}\rho\mathcal{O} - \{\mathcal{O}^2,\rho\}$
- $\mathcal{H}[\mathcal{O}](\rho) = \mathcal{O}\rho + \rho\mathcal{O} - 2\,\mathrm{tr}(\mathcal{O}\rho)\rho$

The corresponding measurement signal verifies:

$$\mathcal{S}_t = \mathrm{tr}(\mathcal{O}\rho_t) + \frac{1}{2}w_t.$$

Formally, dynamical reduction models (like CSL) are just continuous measurements of the mass density in every point of space which is just what is needed.

Standard quantum feedback like computations give:

Final equation

$$\frac{d\rho}{dt} = -i[H + \hat{V}_{\text{pair}}, \rho] + \mathrm{ID} + \mathrm{IC} + \mathrm{GD} + \mathrm{GN}$$

with:

- **Intrinsic decoherence**

$$\mathrm{ID} = \frac{\gamma}{4}\int dx\, \mathcal{D}[\hat{M}_\sigma(x)](\rho),$$

- **Intrinsic collapse**

$$\mathrm{IC} = \frac{\sqrt{\gamma}}{2}\int dx\, \mathcal{H}[\hat{M}_\sigma(x)](\rho)w(x),$$

- **Gravitational decoherence**

$$\mathrm{GD} = \frac{1}{\gamma}\int dx\, \mathcal{D}[\hat{\Phi}(x)](\rho),$$

- **Gravitational noise**

$$\mathrm{GN} = \frac{1}{\sqrt{\gamma}}dx\, \mathcal{H}[i\,\hat{\Phi}(x)](\rho)w(x),$$

the additional notation $\hat{\Phi}(x) = -G\int dy \frac{\hat{M}_\sigma(y)}{|x-y|}$, and:

- **Pair potential**

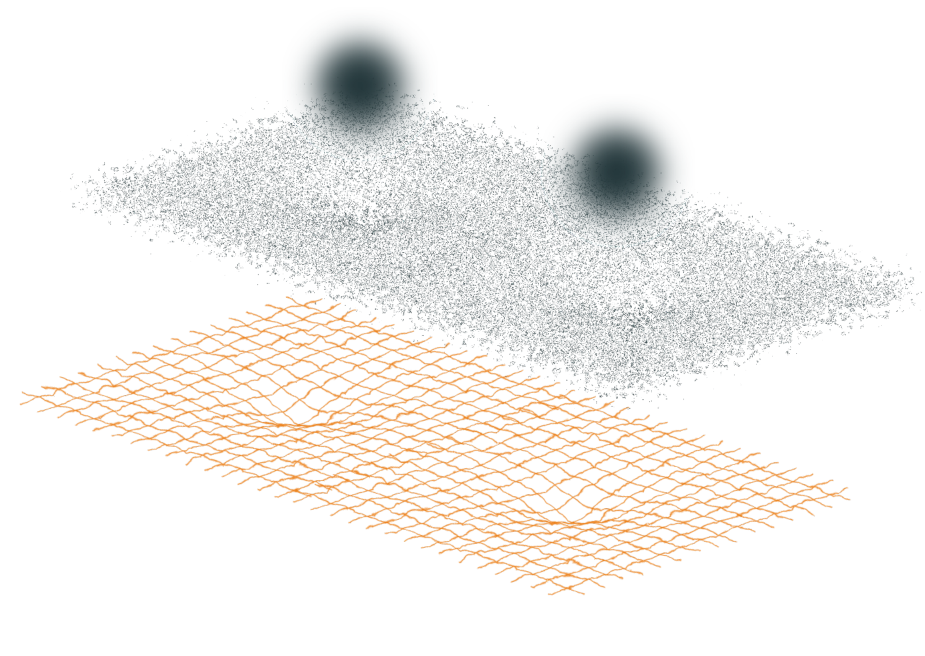
$$\hat{V}_{\text{pair}} = -\frac{1}{2}G\int dx \frac{\hat{M}_\sigma(x)\hat{M}_\sigma(y)}{|x-y|}$$

which only gives rise to a constant self-interaction energy and thus to no 1 particle self-interaction.

Predictions of the theory

1. No faster than light signalling
2. The Born rule holds
3. No one-particle self-interaction
4. Gravitational decoherence **inversely** proportional to intrinsic decoherence → falsifiable (∀ param.)

Status of the "signal": Is it "information" or real stuff? It should be though of as a **real** stochastic field, the **primitive ontology** of the theory, the tangible link between space-time and quantum matter:



In Nature, there are *no* detectors but the fact that this theory is *formally* equivalent to quantum measurement + feedback insures that it is mathematically consistent.

Conclusion

There exists a class of alternatives to the Schrödinger-Newton equation that have nicer fundamental properties. These promising approaches still have to be confronted with upcoming experiments and extended to relativist settings.

References

[1] "*Sourcing semiclassical gravity from spontaneously localised quantum matter*" A. Tilloy, L. Diósi, *Phys. Rev. D* **93** (2016)

[2] "*Probing Gravitational Cat States in Canonical Quantum Theory vs Objective Collapse Theories*", M. Derakhshani, arXiv:1609.01711, (2016)

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