QUANTUM JUMPS FROM CONTINUOUS QUANTUM TRAJECTORIES

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LPTM seminar, Cergy, October 15th, 2015

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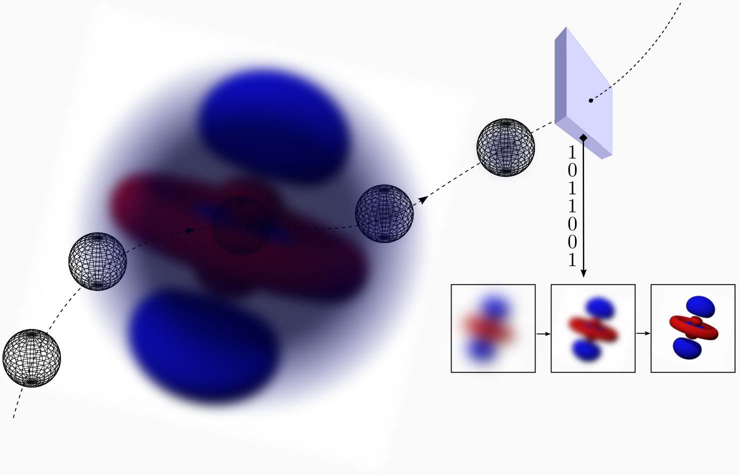
ABOUT



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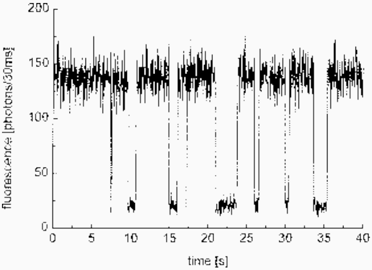
ABOUT

Work done with Denis Bernard and Michel Bauer and mostly based on `arXiv:1410.7231`.

The objective is to understand the emergence of quantum jumps from a finer study of continuous measurements. See quantum jumps as the limit of some more detailed evolution.



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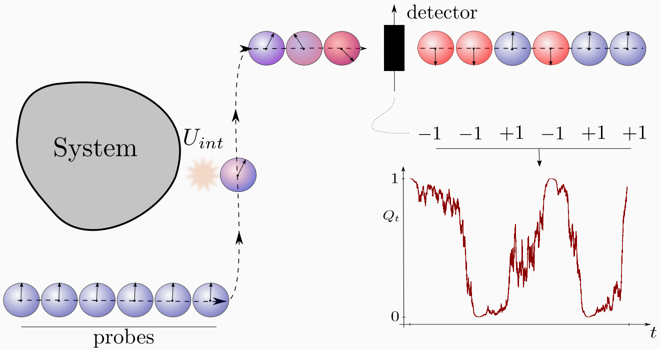
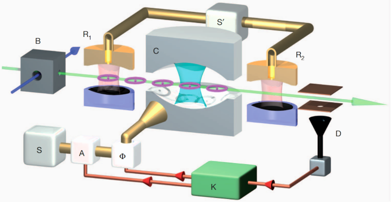
More precisely

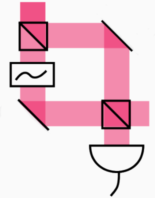


Ideal situations of application

- Discrete situations “a la Haroche”, with actual repeated interactions



- True continuous measurement settings (homodyne detection in quantum optics)



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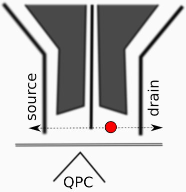
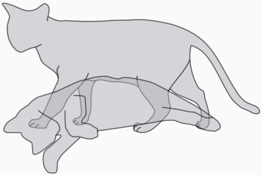
Other applications

- **Any** progressive measurement (e.g. quantum point contacts)



- Dynamical reduction models in foundations (not today)



MODEL

System Hilbert space $\mathcal{H}_s$, "probe" Hilbert space $\mathcal{H}_p = \mathbb{C}^2$. The full density matrix is initially in a product state: $\rho = \rho_s \otimes |+\rangle\langle+|$

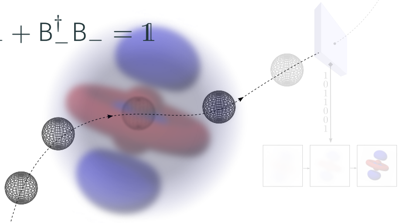
One weak measurement consists in:

1– **Unitary evolution** entangling the system and the probe:

$$\rho \to B_+ \rho_s B_+^\dagger \otimes |+\rangle\langle+| \; + \; B_- \rho_s B_-^\dagger \otimes |-\rangle\langle-|$$

$\sim$ to taking a picture of the particle but not looking at it yet.

Unitarity only implies: $B_+^\dagger B_+ + B_-^\dagger B_- = \mathbb{1}$



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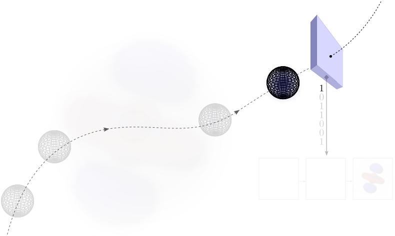
2– **Measurement** of the probe

$$\rho \to \frac{B_{\pm}\rho_s B_{\pm}^{\dagger} \otimes |\pm\rangle\langle\pm|}{\mathrm{tr}(B_{\pm}\rho_s B_{\pm}^{\dagger})} \quad \text{and result } \pm 1$$

$\sim$ to reading the picture and updating the probability



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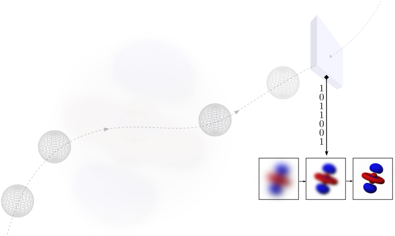
MODEL

3– Forgetting about the probe and taking a **new** one $|+\rangle\langle+|$ for the next iteration



Scaling

Develop B_+ and B_- in the vicinity of $\mathbb{1}/\sqrt{2}$ with the constraint:

$$B_+^\dagger B_+ + B_-^\dagger B_- = \mathbb{1}$$

General solution

$$B_\pm = \frac{1}{\sqrt{2}}\left[1 \pm \sqrt{\epsilon} N_\pm - \epsilon\left(\pm M_\pm + \frac{1}{2} N_\pm^\dagger N_\pm\right) + \mathcal{O}(\epsilon^{3/2})\right]$$

with $\Re\mathrm{e}(N_+) = \Re\mathrm{e}(N_-)$ and $\Re\mathrm{e}(M_+) = \Re\mathrm{e}(M_-)$

See e.g. `arXiv:1303.6658` or `arXiv:1312.1600`

In practice

If we put additional constraints:

- well defined continuous limit
- the interaction with the probe does not change the Hamiltonian of the system

We get:

$$B_{\pm} = \frac{1}{\sqrt{2}}\left[1 \pm \sqrt{\epsilon}N - \frac{\epsilon}{2}N^{\dagger}N + \mathcal{O}(\epsilon^{3/2})\right]$$

where N is just any matrix.

Next steps

- Compute $\mathrm{d}\rho(\mathrm{t}) = \rho(\mathrm{t} + \mathrm{dt}) - \rho(\mathrm{t})$ with $\mathrm{dt} = \epsilon$ explicitly (expand everything up to order dt).
- Separate the random part coming from the measurement in [average] + [noise with zero average] (Doob martingale decomposition)
- Notice that [noise with zero average] becomes white noise in the continuous limit

Result

$$d\rho_t = \underbrace{\left(N\rho_t N^\dagger - \frac{N^\dagger N\rho_t + \rho_t N^\dagger N}{2}\right)}_{L_N(\rho_t)} dt + \underbrace{\left(N\rho_t + \rho_t N^\dagger - \mathrm{tr}\left[N\rho_t + \rho_t N^\dagger\right]\rho_t\right)}_{D_N(\rho_t)} dW_t$$

- $L_N(\rho_t)$ is the Linbladian, responsible for **decoherence**
- $D_N(\rho_t)$ is the responsible for the **collapse**
- W_t is a Wiener process, i.e. dW/dt is white noise (with Itô convention)

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Pure measurement

Take a qubit ($\mathcal{H}_s = \mathbb{C}^2$), $N = \sqrt{\gamma}\,\sigma_z$ and **no** qubit Hamiltonian.

- The phases decrease exponentially fast with characteristic time γ^{-1}
- The probabilities obey:

$$dP_t = 2\sqrt{\gamma}\,P_t(1 - P_t)dW_t$$

  with $P_t = \langle + | \rho_t | + \rangle$ and are decoupled from the phases.

Pure measurement

Focus on the probabilities:

$$dP_t = 2\sqrt{\gamma}\, P_t(1 - P_t) dW_t$$

The SDE has two fixed points, 0 and 1 corresponding to perfect certainty in the eigenbasis of σ_z.

$\longrightarrow$ progressive collapse

Qubit coupled to a thermal bath

Long story short: in a proper limit (weak coupling, infinite bath) probabilities behave as in the classical case:

$$\mathrm{d}P_t = \lambda(p - P_t)\mathrm{d}t$$

where λ is the system-bath coupling and p the equilibrium probability.

$\rightarrow$ exponential convergence to the equilibrium probability p.

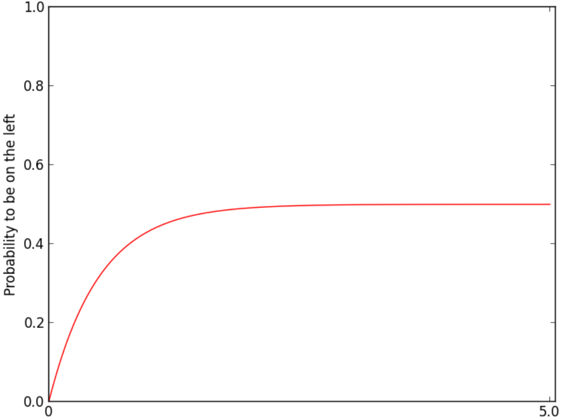
Put the two together!

$$dP_t = \lambda(p - P_t)dt + 2\sqrt{\gamma}\,P_t(1 - P_t)dW_t$$

Non trivial competition between thermalization and information extraction. [studied in `arXiv:1308.0793` by Michel and Denis]

Fascinating equation

Results



No Measurements

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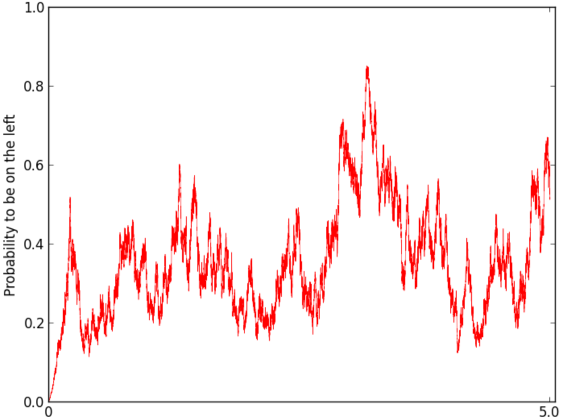
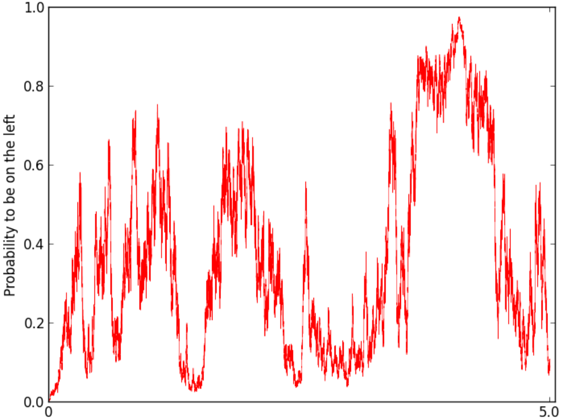
Results



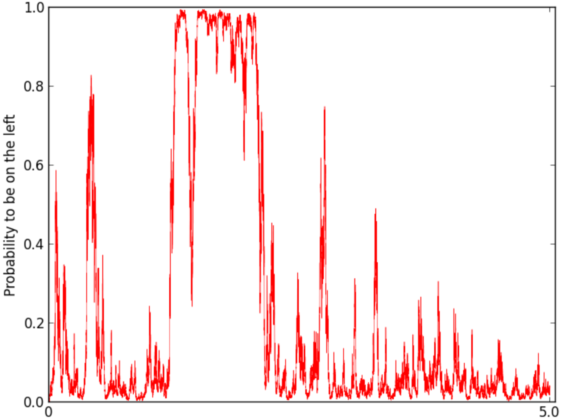
$\gamma = 0.5$

Results



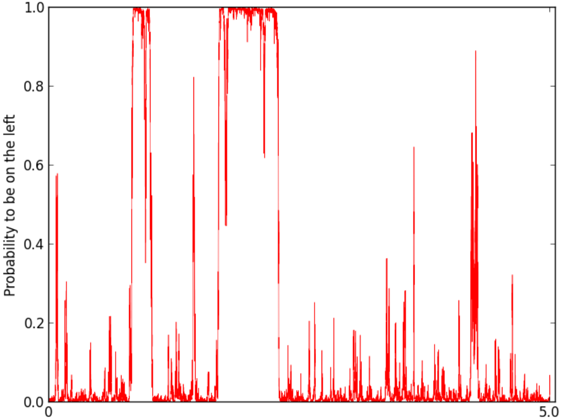
$\gamma = 1.0$

Results



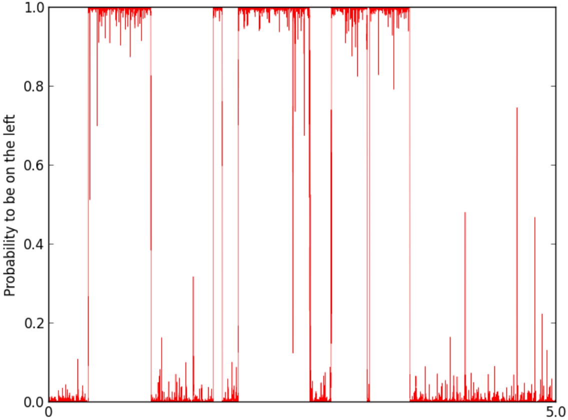
$\gamma = 2.0$

Results



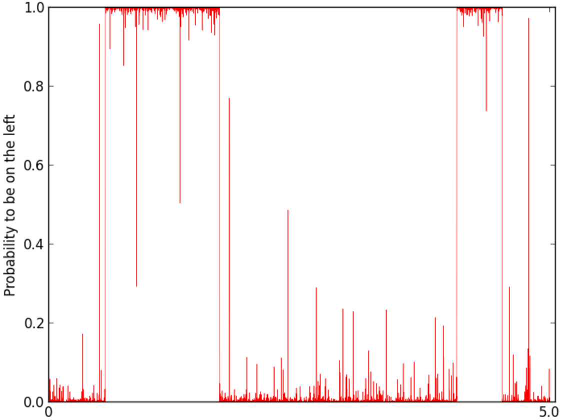
$\gamma = 5.0$

Results



$\gamma = 20.0$

Results



$\gamma = 100.0$, no difference with $\gamma = +\infty$

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Conclusion

A jumpy behavior "emerges". We do not "reveal" an underlying jump process but provide finer continuous description of quantum jumps.

Actually, one can find a hidden variable model for the previous SDE. In this case, we `do` reveal a preexisting jump process (see `arXiv:1510.01232`). The next example will eliminate this possibility

Qubit in an external field

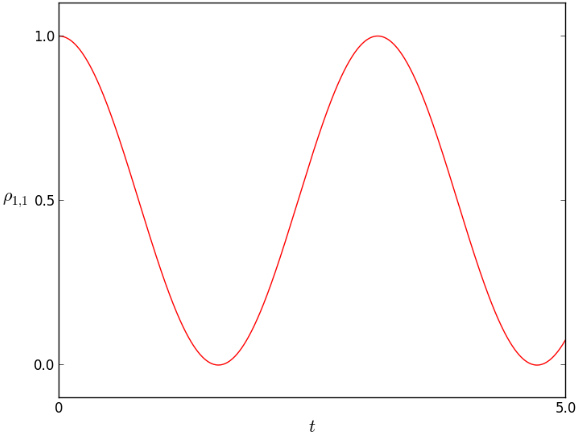
Consider a two level system (a qubit) with Hamiltonian $H = \frac{\omega}{2}\sigma_x$ with σ_z continuously monitored at a rate γ.

The evolution is given by the stochastic master equation:

$$d\rho_t = -i\frac{\omega}{2}[\sigma_x, \rho_t]dt + \underbrace{\gamma L_{\sigma_z}(\rho_t)dt + \sqrt{\gamma}D_{\sigma_z}(\rho_t)dW_t}_{\text{same measurement as before}}$$

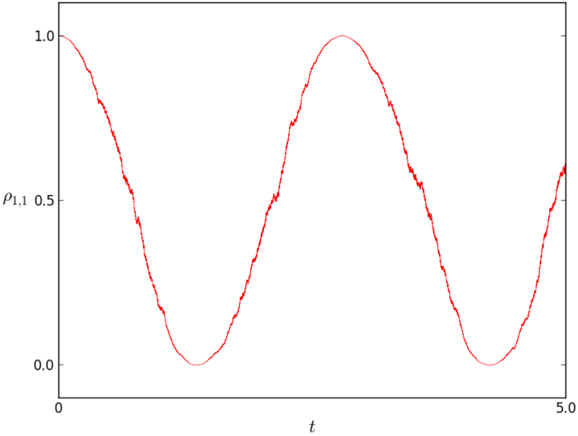
We will look at $\langle +|\rho_t|+\rangle_z$, i.e. at the probabilities in the eigenbasis of the measurement.

Results



Without measurement $\gamma = 0.0$

Results



$\gamma = 0.1$

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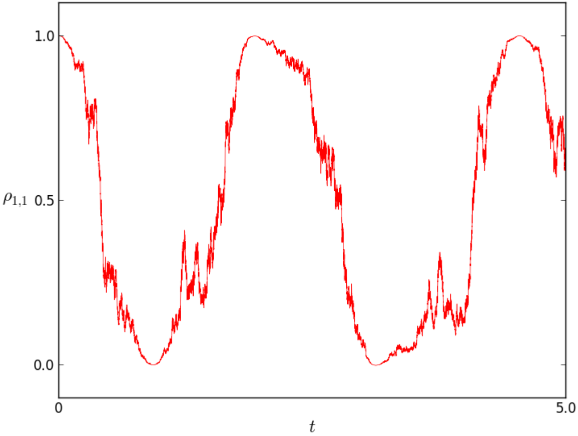
Results



$\gamma = 0.5$

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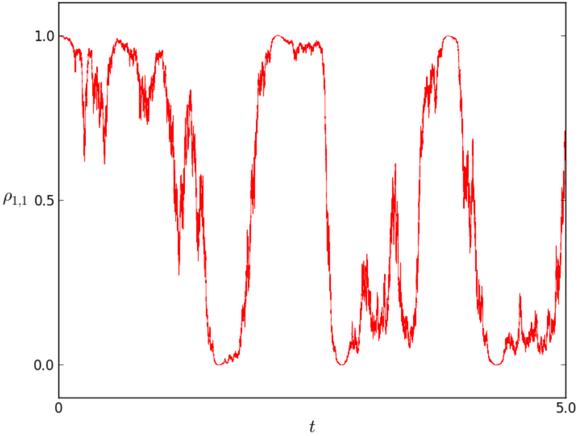
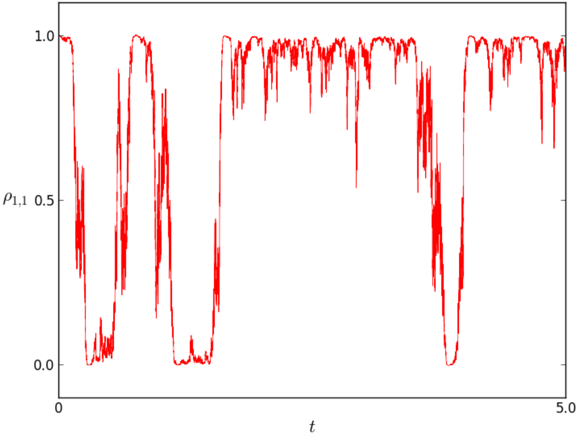
Results



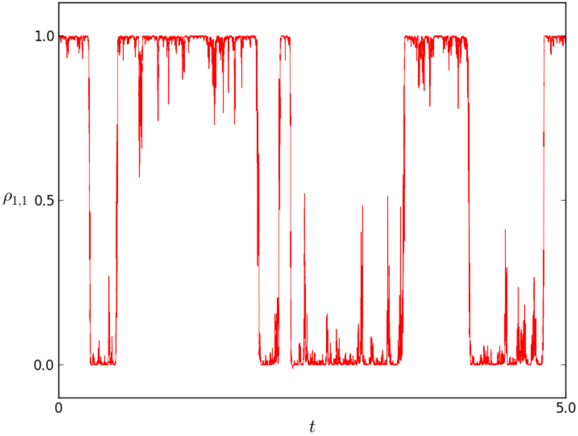
$\gamma = 1.0$

Results



$\gamma = 2.0$

Results



$\gamma = 5.0$

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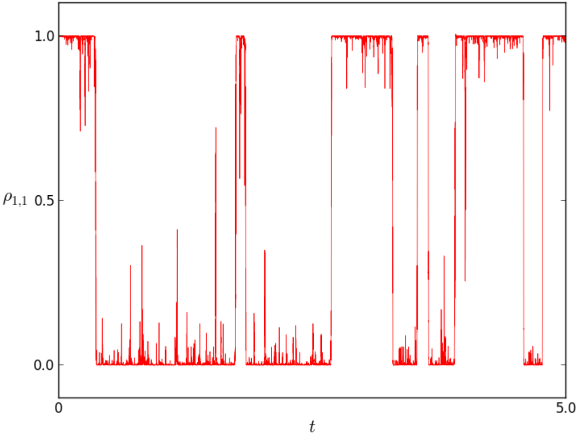
Results



$\gamma = 10$

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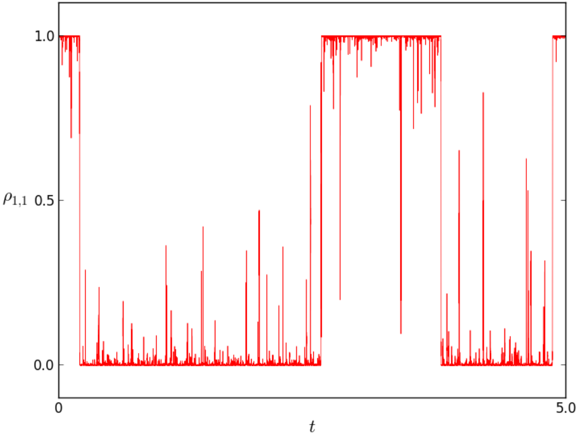
Results



$\gamma = 20$

Results

Actually, I had to cheat a bit and take $\omega \propto \gamma$ for the previous plots to counter the Zeno effect.

Theorem

Consider quantum system subjected to the measurement of the operator $\mathcal{O}$ at rate γ and with an evolution without measure:

$$\frac{d\rho_t}{dt} = \mathcal{L}(\rho_t) = -i[H, \rho_t] + L_M(\rho_t)$$

1. For large γ its density matrix ρ behaves like a continuous time Markov chain between the eigenvectors of $\mathcal{O}$
2. The jump rates m_{ij} can be computed exactly as a function of $\mathcal{L}$ and $\mathcal{O}$. The generic form is a bit complicated but the dominant contribution if of the form :

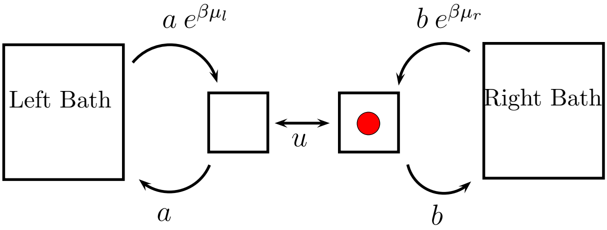
$$m_{ij} \propto \underbrace{\frac{[\text{coeff. of H}]^2}{\gamma}}_{\text{Zeno effect!}} + \text{coeff. of } L_M$$

Comments

- The convergence is weak in the sense that it is only valid for the finite dimensional distributions (spikes don't disappear)
- The generalization to multiple observables is easy
- The measurement efficiency does not matter (i.e. you can "miss" probes without changing the formulae)
- The Zeno effect does not touch the jumps induced by the coupling with a bath

Possible application

Exploit the different behavior of unitary quantum jumps and thermal quantum jumps with respect to the Zeno effect to control systems `arXiv:1404.7391`



$\rightarrow$ Maxwell Daemon from measurement only!

Idea of the proof

Not completely standard because strong noise limit $\longrightarrow$ "perturb" around the pure measurement situation

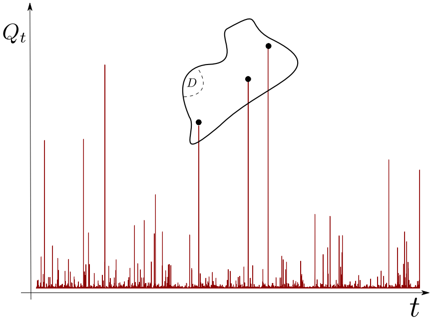
- Consider the probability kernel $K_t(\rho_0, d\rho)$ to go from a given density matrix ρ_0 to another density matrix ρ, up to $d\rho$, after a time t.
- Write its Kolmogorov equation $\partial_t K = K\mathfrak{D}$ where $\mathfrak{D}$ can be expanded in:

$$\mathfrak{D} = \gamma^2 \mathfrak{D}_2 + \mathfrak{D}_0$$

- Compute the eigenvectors of $\mathfrak{D}_2$ (invariant measures) and perturbatively expand $K_t = e^{t\gamma^2 \mathfrak{D}_2 + t\mathfrak{D}_0}$

Spikes

Sharp scale invariant fluctuations, “**spikes**”, decorate the jump process when $\gamma \to +\infty$.



This implies that the convergence is necessarily weak.

What could be done?

- Study the fluctuations, the spikes, in the general case (specific cases already studied in `arXiv:1510.01232`).
- Study the infinite dimensional setting (already some earlier study in the context of dynamical reduction models by Bassi and Dürr)
- Probe the semi-classical behavior of many systems! (tunneling processes, trajectories in cloud chambers, etc.)
- Apply the technique of the proof to other strong noise systems (turbulence?)

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More generally

Repeated interactions have applications in:

- Quantum information
- Quantum control
- Quantum foundations
- Stochastic Thermodynamics