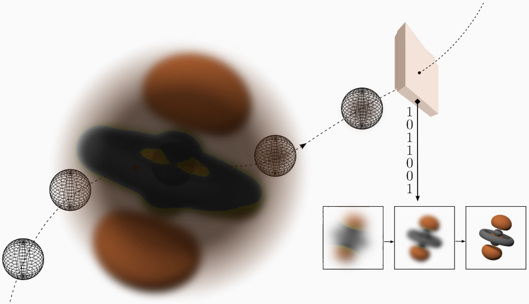
The common structure of “heterodox” collapse models, “orthodox” continuous quantum measurement and classical stochastic filtering



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February 2, 2017

Why is there a problem?

"We know that the moon is demonstrably not there when nobody looks"

David Mermin 1981

The only connexion between the **formalism** of quantum theory and **Nature** is through the measurement postulate.

"A **mathematically trivial** operation"

Measurement postulate

For a system "described" by $|\psi\rangle \in \mathscr{H}$ and a measurement of orthogonal projectors Π_i s. t. $\sum_i \Pi_i = \mathbb{1}$ one has:

Born rule :

Result " i " with probability $\mathbb{P}[i] = \langle\psi|\, \Pi_i \,|\psi\rangle$

Collapse :

$$|\psi\rangle \longrightarrow \frac{\Pi_i \,|\psi\rangle}{\sqrt{\mathbb{P}[i]}}$$

Max Born 1926

John von Neumann 1932

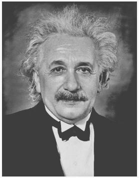
"A **physically subtle** endeavour"

- What is a measurement?
- How can measurement be a primitive concept?
- What is a measurement result made of?
- Can one deduce the measurement postulate from unitary evolution? (answer: NO, hint: decoherence does not help)

notions of 'reversible' and 'irreversible'. Einstein said that it is theory which decides what is 'observable'. I think he was right — 'observation' is a complicated and theory-laden business. Then that notion should not appear in the *formulation* of fundamental theory. *Information? Whose* information? Information about *what*?

On this list of bad words from good books, the worst of all is 'measurement'. It must have a section to itself.

Physics World, Against Measurement



Albert Einstein 1935

John S. Bell 1989

The curse of linearity in a “measurement” situation

Initial state:

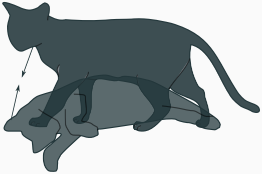
$$|\Psi\rangle_0 = \frac{1}{\sqrt{2}}\Big(\,|\uparrow\rangle + |\downarrow\rangle\,\Big) \otimes |\spadesuit\rangle \quad \in \mathbb{C}^2 \otimes \mathbb{C}^{2^{10^{23}}}$$

After evolution:

$$|\Psi\rangle_t = \frac{1}{\sqrt{2}}\,|\uparrow\rangle \otimes |\diamondsuit\rangle + \frac{1}{\sqrt{2}}\,|\downarrow\rangle \otimes |\heartsuit\rangle$$

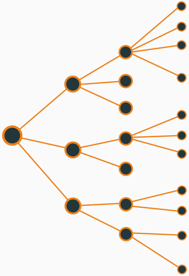
$\rightarrow$ typically $|\clubsuit\rangle \perp |\heartsuit\rangle$: **decoherence**

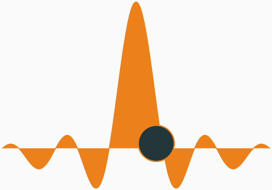
$\rightarrow$ but **no collapse** because of linearity

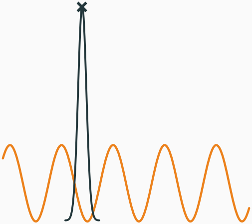


INTRODUCTION

Reformulations of quantum theory are needed







Everett 1957

Bohm 1952

Ghirardi 1986

OUTLINE

- The pre-Genesis of collapse models
- **Repeated and continuous orthodox measurements**
- The parallel with stochastic filtering
- Back to collapse
- Open problems

PRE-GENESIS

Objective

Find a modification of the Schrödinger equation that:

- Does almost nothing to microscopic dynamics
- Collapses macroscopic superpositions
- Does so according to the Born rule

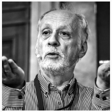
MAIN LESSON

To avoid **faster than light signaling**, the evolution needs to be linear at the master equation level

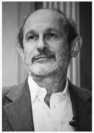
Defining $\bar{\rho}_t = \mathbb{E}[\rho_t]$ one needs to have:

$$\bar{\rho}_t = \Phi_t \cdot \bar{\rho}_0$$

with Φ_t Completely Positive Trace Preserving
Asking for Markovianity $\rightarrow$ Lindblad form



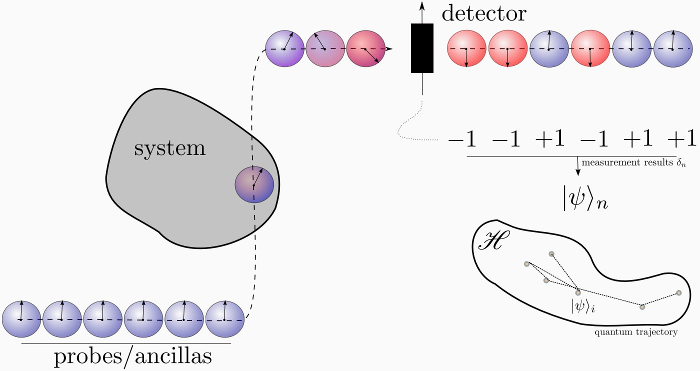
Gisin



Diósi

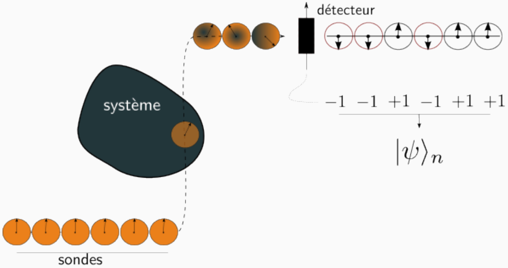
REPEATED AND CONTINUOUS MEASUREMENTS

REPEATED INTERACTION SCHEMES



Situation considered

- System, $|\psi\rangle \in \mathscr{H}_s$
- Probe, $\mathscr{H}_p = \mathbb{C}^2$
- Unitary interaction
- Measurement of σ_z on the probe



REPEATED INTERACTIONS

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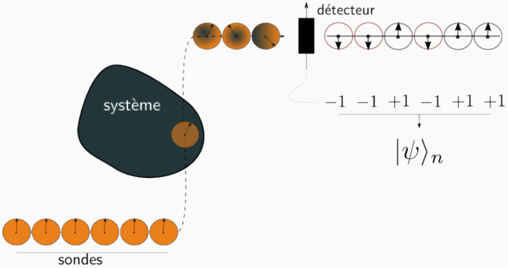


$$|\psi\rangle_n \otimes |+\rangle_x \xrightarrow{\text{interaction}} \hat{\Omega}_+ |\psi\rangle_n \otimes |+\rangle_z + \hat{\Omega}_- |\psi\rangle_n \otimes |-\rangle_z$$

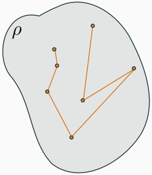
$$\xrightarrow{\text{mesurement}} |\psi\rangle_{n+1} = \frac{\Omega_\pm |\psi\rangle_n}{\sqrt{\langle\psi| \Omega_\pm^\dagger \Omega_\pm |\psi\rangle_n}}$$

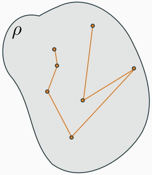
with the **only** constraint:

$$\Omega_+^\dagger \Omega_+ + \Omega_-^\dagger \Omega_- = \mathbb{1}$$

Discrete quantum trajectories

A sequence $|\psi\rangle_n$ or ρ_n (random) and the corresponding measurement results $\delta_n = \pm 1$.





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$\Rightarrow$ Make the interactions soft and frequent:

$$\Omega_\pm = \frac{1}{\sqrt{2}}\left(\mathbb{1} \pm \mathcal{O}\,\varepsilon + \#\,\varepsilon^2 + \cdots\right)$$

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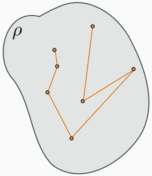

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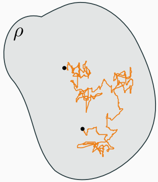
Continuous quantum trajectories

A continuous process $|\psi\rangle_t$ or ρ_t (random) and the corresponding measurement signal y_t:

$$y_t \propto \sqrt{\Delta t} \sum_{n=1}^{t/\Delta t} \delta_n$$



RESULT

Stochastic master equation (∼ 1987)

State (density matrix or pure state):

$$\mathrm{d}\rho_t = \mathscr{L}(\rho_t)\,\mathrm{d}t + \gamma\,\mathcal{D}[\mathcal{O}](\rho_t)\,\mathrm{d}t + \sqrt{\gamma}\,\mathcal{H}[\mathcal{O}](\rho_t)\,\mathrm{d}W_t$$

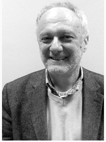
Signal:

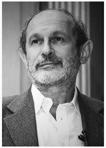
$$\mathrm{d}y_t = \sqrt{\gamma}\,\mathrm{tr}\left[\left(\mathcal{O} + \mathcal{O}^\dagger\right)\rho_t\right]\,\mathrm{d}t + \mathrm{d}W_t$$

with:

- $\mathcal{D}[\mathcal{O}](\rho) = \mathcal{O}\rho\mathcal{O}^\dagger - \frac{1}{2}\left(\mathcal{O}^\dagger\mathcal{O}\rho + \rho\mathcal{O}^\dagger\mathcal{O}\right)$
  «decoherence and dissipation»
- $\mathcal{H}[\mathcal{O}](\rho) = \mathcal{O}\rho + \rho\mathcal{O}^\dagger - \mathrm{tr}\left[\left(\mathcal{O} + \mathcal{O}^\dagger\right)\rho\right]\rho$
  «aquisition of information»
- $\frac{\mathrm{d}W_t}{\mathrm{d}t}$ white noise


V. Belavkin


A. Barchielli


L. Diósi

EXAMPLE

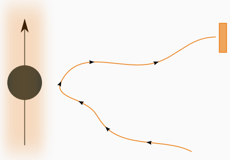
Situation considered

"Pure" continuous measurement of a qubit

Qubit $\Rightarrow \mathscr{H} = \mathbb{C}^2$ so $\rho_t = \begin{pmatrix} p_t & u_t \\ u_t^* & 1 - p_t \end{pmatrix}$

Continuous energy measurement, i.e.

$\mathcal{O} = \sigma_z \propto H$



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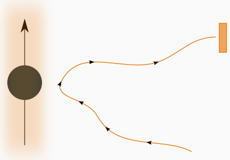

Equation for the population

$$\mathrm{d}p_t = \sqrt{\gamma}\, p_t(1 - p_t)\, \mathrm{d}W_t$$

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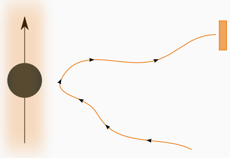


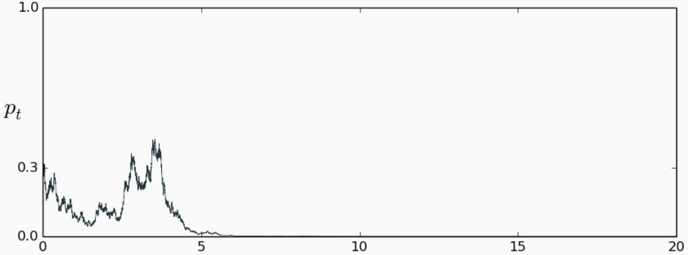
Equation for the population

$$\mathrm{d}p_t = \sqrt{\gamma}\, p_t(1 - p_t)\,\mathrm{d}W_t$$

Equation for the phase

$$\mathrm{d}u_t = -\frac{\gamma}{8} u_t\,\mathrm{d}t + \frac{\sqrt{\gamma}}{2}(2p_t - 1)\,\mathrm{d}W_t$$

"Pure" continuous measurement of a qubit



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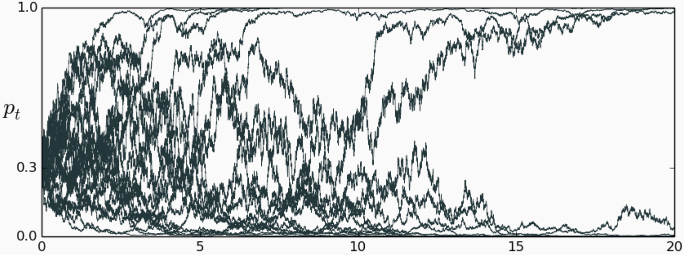

BORN RULE

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so finally:

$$\mathbb{P}[p_t \to 1] = p_0$$

LOCAL CONCLUSION

→ Collapse now has a timescale γ^{-1}

→ The Born rule stays valid

→ The trajectory is real

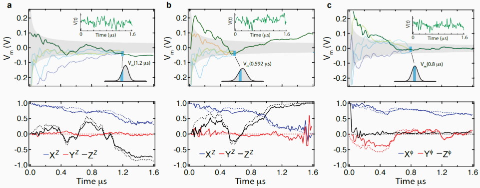

FIG. 3: Quantum trajectories. **a,b** Individual measurement traces obtained for Z-measurements with $\bar{n} = 0.4$. The top panel displays $V_m(\tau)$ as a green line, with the inset displaying the instantaneous measurement voltage. The gray region indicates the standard deviation of the distribution of measurement values. Measurement traces that converge to an integrated value within the blue matching window are used to tomographically reconstruct the trajectory at that time point. A few different measurement traces that contribute to the reconstruction at 1.2 μs (**a**) and 0.592 μs (**b**) are indicated in pastel colors. The lower insets indicate the distribution of measurement values with the matching window indicated in blue. Quantum trajectories obtained from analysis of the measurement signal are shown as dashed lines in the lower panel. Solid lines indicate the tomographically reconstructed quantum trajectory based on the ensemble of measurements that are within the matching window of the original measurement signal. **c** Individual measurement traces and associated quantum trajectory obtained for a ϕ-measurement with $\bar{n} = 0.4$.

Quantum trajectories from the group of Irfan Siddiqi at Berkeley, Nature **502**, 211 (2013)

ONE EXAMPLE OF ITÔ WITHOUT FEEDBACK

How fast do we purify?

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Look at $\Delta_t = \sqrt{\det(\rho_t)} = \sqrt{p_t(1-p_t)}$ and compute $\mathrm{d}\Delta_t$

$$\mathrm{d}\Delta_t = -\underbrace{\frac{\gamma}{8}\Delta_t\,\mathrm{d}t}_{\text{Itô correction}} + \frac{1}{2}\sqrt{\gamma\Delta_t}(1-2p_t)\,\mathrm{d}W_t$$

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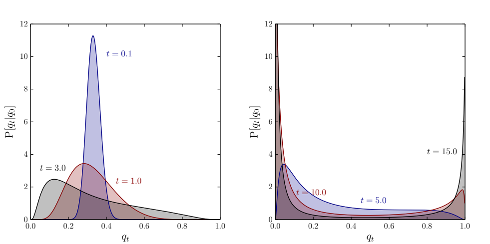
Let us look at the average $\bar{\Delta}_t = \mathbb{E}[\Delta_t]$

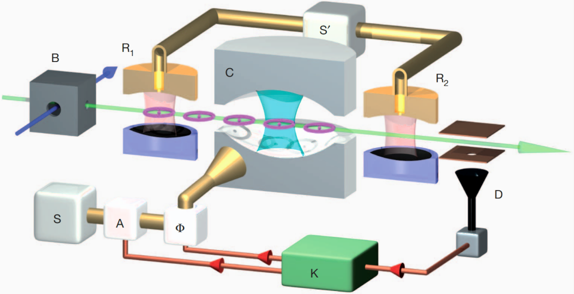
$$\frac{\mathrm{d}\bar{\Delta}_t}{\mathrm{d}t} = -\frac{\gamma}{8}\bar{\Delta}_t \quad \Rightarrow \quad \bar{\Delta}_t = \bar{\Delta}_0\, e^{-\frac{\gamma t}{8}}$$

OTHER WAYS?

$$\mathrm{d}\mathbb{P}[p_t = p|p_0] = \frac{2p_0}{\sqrt{2\pi\gamma t}p(1-p)} \exp\left[-\frac{\left(\frac{2}{\sqrt{\gamma}}\left(\ln\left[\frac{p}{1-p}\right] - \ln\left[\frac{p_0}{1-p_0}\right]\right) - \sqrt{\gamma}t/2\right)^2}{2t}\right]\mathrm{d}p$$
$$+ \frac{2(1-p_0)}{\sqrt{2\pi\gamma t}p(1-p)} \exp\left[-\frac{\left(\frac{2}{\sqrt{\gamma}}\left(\ln\left[\frac{p}{1-p}\right] - \ln\left[\frac{p_0}{1-p_0}\right]\right) + \sqrt{\gamma}t/2\right)^2}{2t}\right]\mathrm{d}p$$

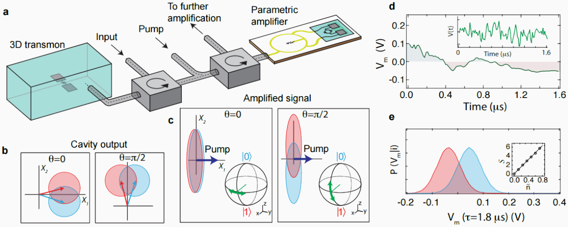


With discrete probes



Gleyzes et al. Nature **446**, 297-300 (2007)

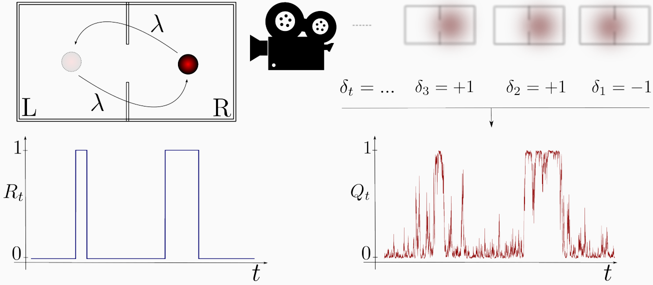
With “continuous” probes



Experimental setup of the group of Irfan Siddiqi at Berkeley, Nature **502**, 211 (2013)

THE PARALLEL WITH STOCHASTIC FILTERING

REPEATED BLURRY CLASSICAL MEASUREMENTS



Bayesian updating of the probability for the particle to be in one side of the box.

Kushner-Stratonovich filtering equation:

Diagonal probability matrix:

$$\mathrm{d}\rho_t = \mathscr{L}(\rho_t)\,\mathrm{d}t + \gamma\,\mathcal{D}[\mathcal{O}](\rho_t)\,\mathrm{d}t + \sqrt{\gamma}\,\mathcal{H}[\mathcal{O}](\rho_t)\,\mathrm{d}W_t$$

Signal:

$$\mathrm{d}y_t = \sqrt{\gamma}\,\mathrm{tr}\left[\left(\mathcal{O} + \mathcal{O}^\dagger\right)\rho_t\right]\,\mathrm{d}t + \mathrm{d}W_t$$

with:

- $\mathcal{D}[\mathcal{O}](\rho) = \mathcal{O}\rho\mathcal{O}^\dagger - \frac{1}{2}\left(\mathcal{O}^\dagger\mathcal{O}\rho + \rho\mathcal{O}^\dagger\mathcal{O}\right)$
- $\mathcal{H}[\mathcal{O}](\rho) = \mathcal{O}\rho + \rho\mathcal{O}^\dagger - \mathrm{tr}\left[\left(\mathcal{O} + \mathcal{O}^\dagger\right)\rho\right]\rho$

BACK TO COLLAPSE

Very weakly measure the mass density of quantum matter everywhere in space:

$$\mathrm{d}\rho_t = -i[H, \rho]\mathrm{d}t - \frac{\gamma}{4} \int_{\mathbb{R}^3} \mathrm{d}\mathbf{x}\, \mathcal{D}[\hat{M}_\sigma(\mathbf{x})](\rho)\mathrm{d}t + \frac{\sqrt{\gamma}}{2} \int_{\mathbb{R}^3} \mathrm{d}\mathbf{x}\, \mathcal{H}[\hat{M}_\sigma(\mathbf{x})](\rho)\mathrm{d}\, W_t(\mathbf{x})$$

Take this as a **fundamental** equation

Theorem

All continuous Markovian collapse models can be written as the continuous measurement of something

OPEN PROBLEMS

IN THE ORTHODOX MEASUREMENT CONTEXT

- Construct a “real-time” continuous non-Markovian measurement theory
- Make tractable models of non-Markovian feedback
- Find good equivalents of forward-backward estimation

IN THE COLLAPSE CONTEXT

- Understand the behavior of nastier collapse models
- Use collapse models to solve gravity related difficulties (cosmological fluctuations, back hole information paradox...)
- Make collapse models fully relativistic without breaking their core features

A FEW THINGS I STUDIED

Measurement context:

- Competition between continuous measurement and evolution
  $\rightarrow$ emergence of quantum jumps and **spikes**
- Optimal information extraction problems
- Control via continuous measurement only
- Hidden variable models for quantum measurement

Collapse context:

- Newtonian quantum gravity in a collapse context
- Monte-Carlo method to plot the realizations of non-Markovian collapse models
- Collapse models for Quantum Field Theory new!