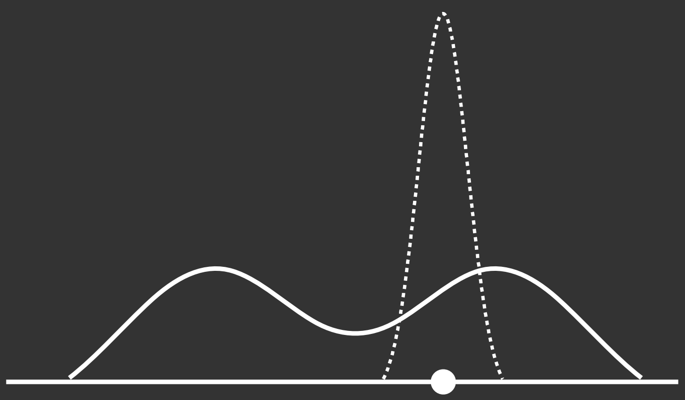
Hiding a collapse mechanism inside the standard model

a new way to construct collapse models

Antoine Tilloy

Max-Planck-Institut für Quantenoptik,
Garching, Germany



ZiF

For: **The Message of Quantum Science II**
"How much have we learned in the past five years"
ZIF Workshop, Universität Bielefeld, 6-10 November 2017

Alexander von Humboldt
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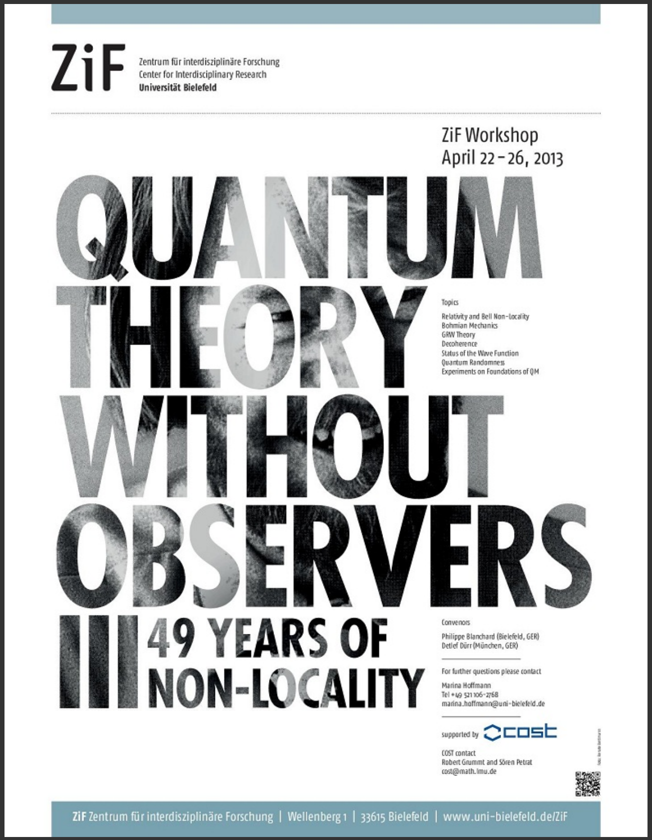
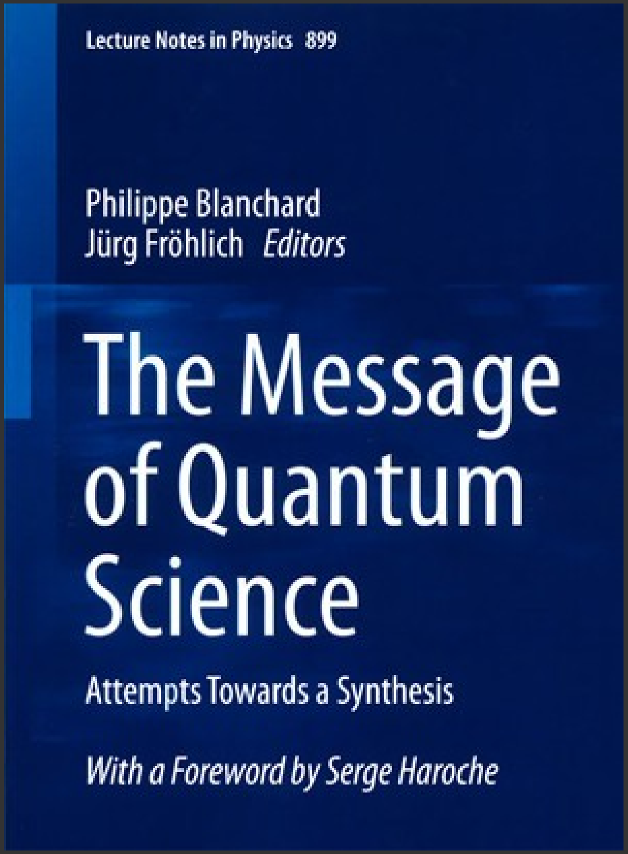
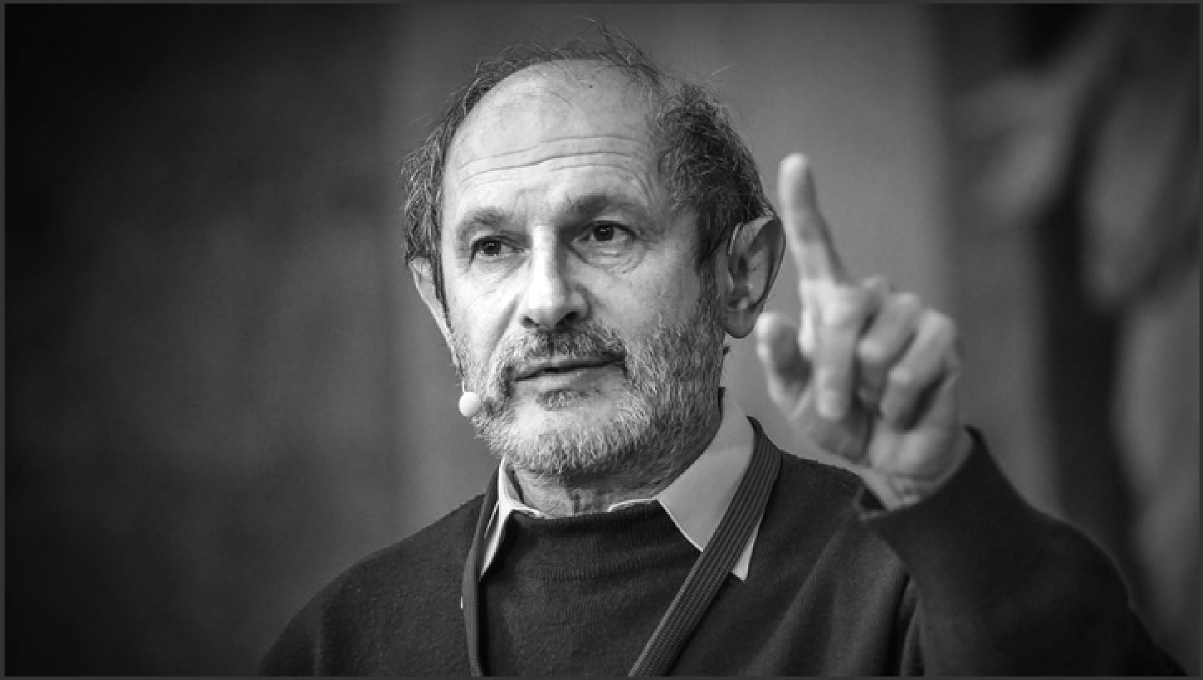
MPQ

Two related great workshops







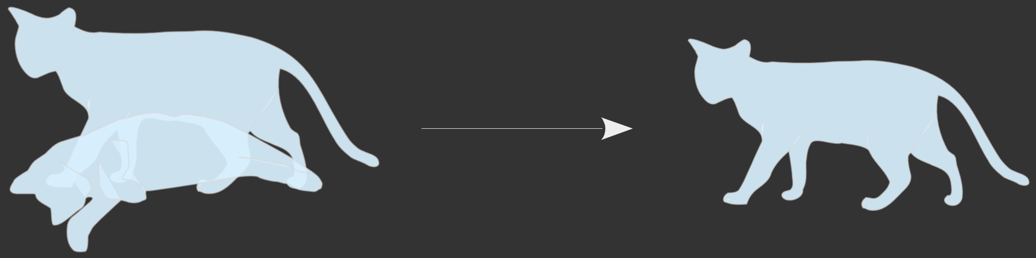
Lajos Diósi

What are collapse models?

Naive definition:

Collapse models are an attempt to solve the measurement problem of quantum mechanics through an *ad hoc*, non-linear, and stochastic modification of the Schrödinger equation.

$$\partial_t |\psi_t\rangle = -iH|\psi_t\rangle + \varepsilon\, f_\xi(|\psi\rangle)$$



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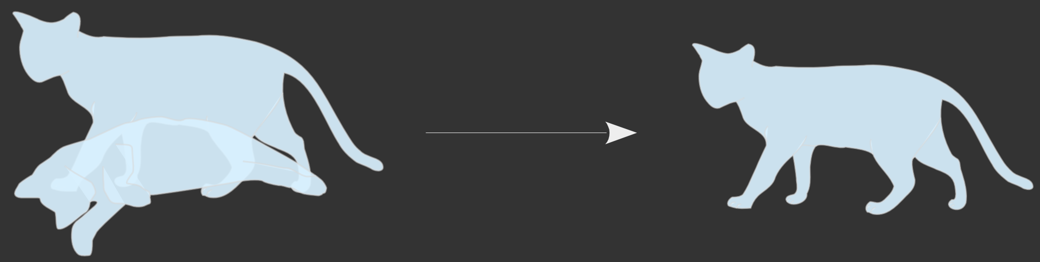

A few names:

Pearle, Ghirardi, Rimini, Weber, Diósi, Adler, Gisin, Tumulka, Bedingham, Penrose, Percival, Bassi, Ferialdi, Weinberg ...

Timeline:

1970 first attempts
1984 first consistent equation (Gisin)
1986 Ghirardi-Rimini-Weber model
1990 abstract idea for QFT (Diósi)
2007 relativistic GRW (Tumulka)
2011 relativistic CSL (Bedingham)

Objective of this talk

Present a new way to construct collapse models that:

– Naturally extends them to **quantum field theory**

– Makes them **empirically indistinguishable** from orthodox QM

Possibly interesting whether or not you like collapse models!

Outline

Introduction to collapse: the **Ghirardi-Rimini-Weber model** (1986)

GRW Model for N spinless particles

- Standard linear evolution between jumps

$$\partial_t|\psi_t\rangle = -iH|\psi_t\rangle$$

- Jump hitting particle *k* in x_f at a rate λ

$$|\psi_t\rangle \to \frac{\hat{L}_k(x_f)|\psi_t\rangle}{\|\hat{L}_k(x_f)|\psi_t\rangle\|}$$

with $P(x_f) = \|\hat{L}_k(x_f)|\psi_t\rangle\|^2$

and $\hat{L}_k(x_f) = \frac{1}{(\pi r_c^2)^{3/2}} \mathrm{e}^{(\hat{x}_k - x_f)^2/(2r_c^2)}$

Ghirardi, G. C., Rimini, A., & Weber, T. (1986) *Phys. Rev.* D, **34**(2), 470.

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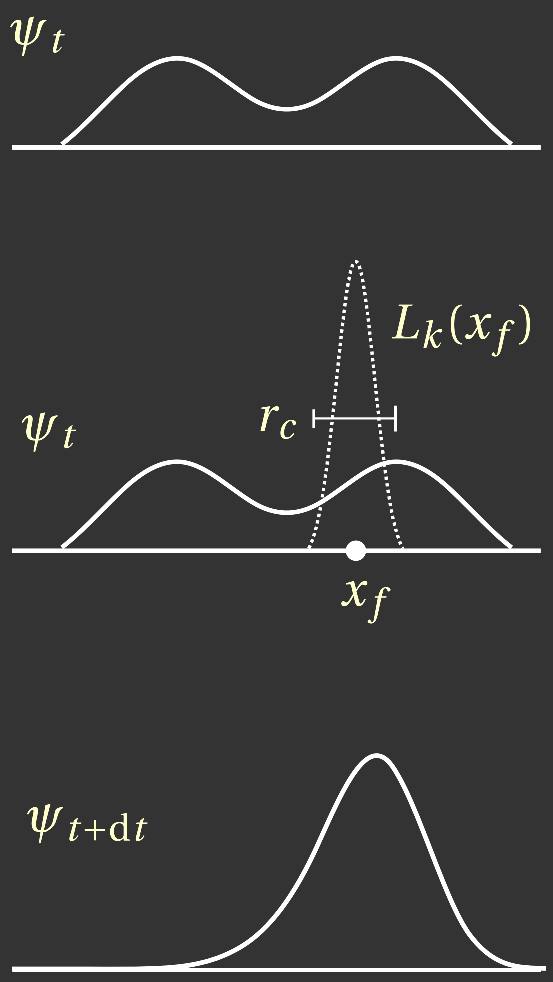

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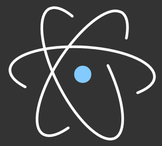
The **Ghirardi-Rimini-Weber model** (1986)

Two new parameters λ and r_c such that:

Weak collapse

A single particle slowly collapses in the position basis

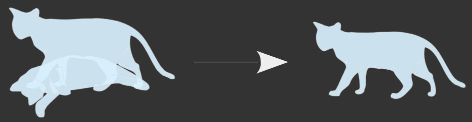
- Microscopic dynamics unchanged



Amplification

The effective collapse rate is renormalized for macroscopic superpositions so that

- Macroscopic superpositions suppressed



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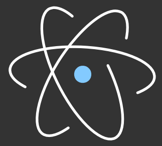

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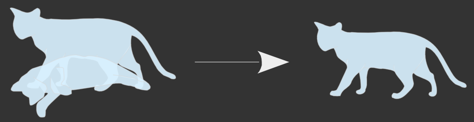

Two questions:
- What is the theory about?
- What does the theory predict?

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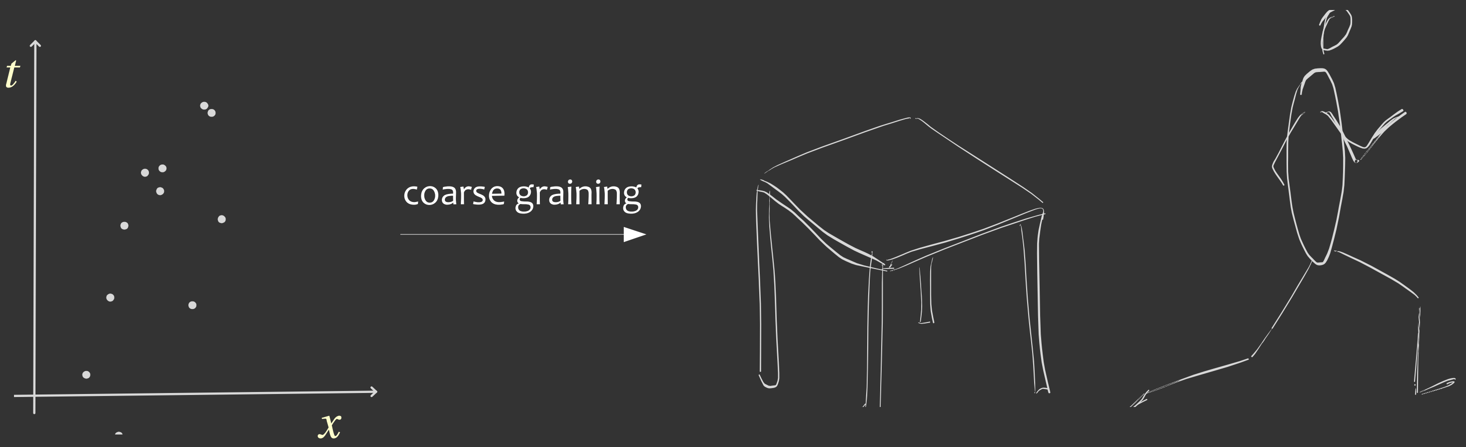
The **Ghirardi-Rimini-Weber model** (1986)

Q1: What is the theory about?

It is about **stuff**
(aka "local beables" aka "primitive ontology")

2 simple options:

- Collapse space-time events or "**flashes**": (x_f, t_f)
- Mass density field: $\langle\psi_t|M(x)|\psi_t\rangle$



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AT, L Diósi, *Phys. Rev. D* **93** (2), 024026
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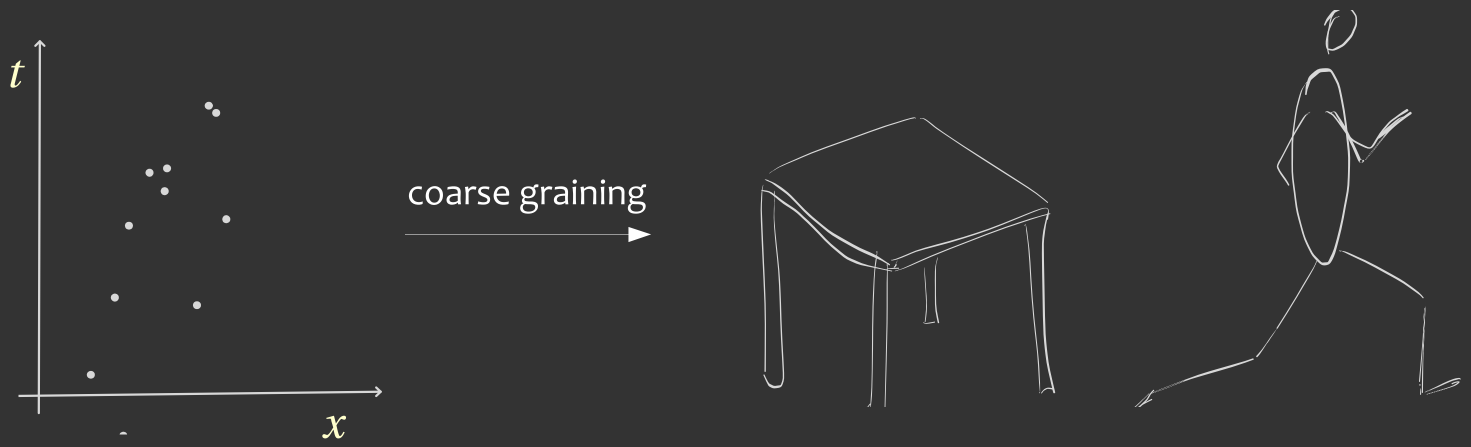

1. Avoids most conceptual problems (like the "tail" problem)
2. Guides unification (especially with gravity)

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The **Ghirardi-Rimini-Weber model** (1986)

Q2: What does the theory predict?

Master equation

Define: $\rho_t = \mathbb{E}\big[|\psi_t\rangle\langle\psi_t|\big]$

$$\partial_t \rho_t = -\frac{i}{\hbar}[H, \rho_t] + \lambda \sum_{k=1}^{N} \int \mathrm{d}x_f \hat{L}_k(x_f)\rho_t \hat{L}_k(x_f) - \rho_t$$

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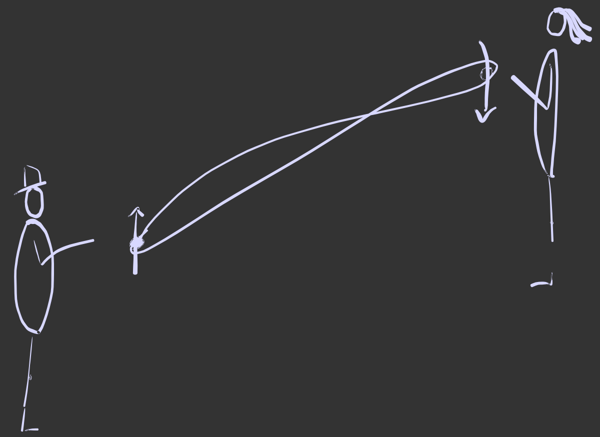
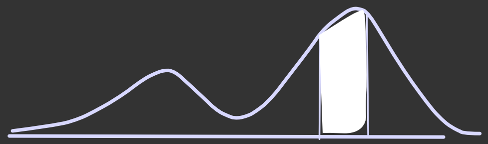
It is:

- linear
- of the Lindblad form



This prevents:

- faster than light signaling
- break down of the Born rule



$\mathbb{P}[x] = |\psi(x)|^2$

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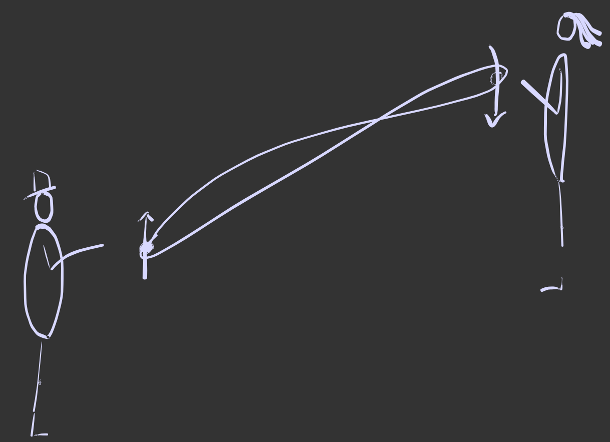

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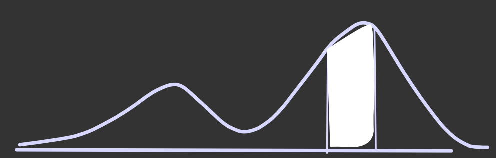

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But it is a disappointment.
GRW is empirically embeddable
in orthodox QM

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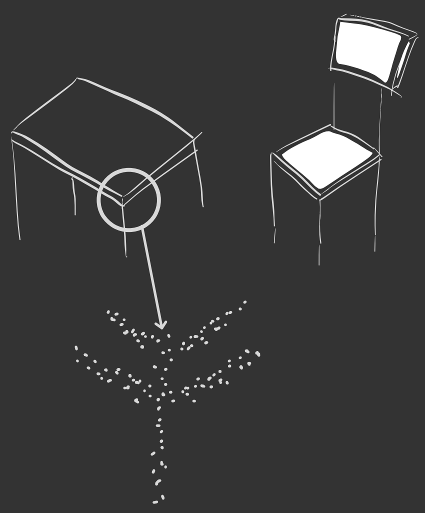
3 levels of analysis

Ontological content

"What the theory says the world is like"

$$(x_f, t_f)$$



State vector (?)

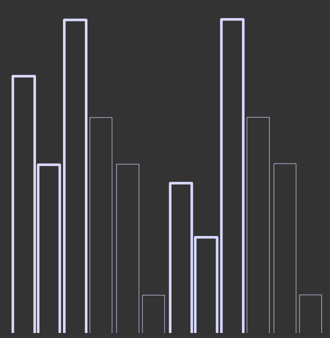
"An intermediary object in the theory"

$$\partial_t |\psi_t\rangle = -iH|\psi_t\rangle + \varepsilon f_\xi(|\psi\rangle)$$

Empirical content

"What the theory predicts"

$$\partial_t \rho_t = \mathscr{L}(\rho_t)$$



More general collapse models

Standard method:

1. Start from some stochastic Schrödinger equation

$$\partial_t |\psi_t\rangle = -iH|\psi_t\rangle + \varepsilon f_\xi(|\psi\rangle)$$

2. Ask that the density matrix is given by a legitimate *CPTP* map

$$\rho_t = \Phi_t \cdot \rho_0$$

→ puts constraints on $f_\xi(|\psi\rangle)$

3. (not always discussed) Define local beables

e.g.: $\langle\psi|\hat{M}(x)|\psi\rangle$

Diósi, L. (1989), Phys. Rev. A, **40**(3), 1165.
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examples:
- CSL
- QMUPL
- Diosi-Penrose
- dissipative CSL
- non-Markovian CSL
- n-M dissipative CSL...

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Local summary

Collapse models propose a solution of the measurement problem.
They have important features:

1. They modify the predictions of the Standard Model but are still empirically equivalent to a quantum evolution on a bigger space

2. The stochastic state used to define them is **not** central
   → the empirical content is in the master equation for ρ_t
   → the metaphysics is in local beables e.g. (x_f, t_f)

3. The important constraint is the linearity at the master equation level, needed to preserve the operational quantum toolbox

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Opinion:
Useful as a tool to construct unification toy models,
Useful historically (precursor to continuous measurement theory)
More?

What are the difficulties?

1. The non-linear modifications of the Schrödinger equation are painfully **ad hoc** (where does it come from? gravity?).

2. It is unclear if there exists generic (sufficiently model-independent) experimental signatures of collapse.

3. Relativistic extensions are difficult in the "stochastic state" representation.

We are going to "solve" these difficulties.

An alternative construction of collapse models

Main idea: construct collapse models the other way around.

Standard way:

$$\partial_t|\psi_t\rangle = -iH|\psi_t\rangle + \varepsilon f_\xi(|\psi\rangle) \quad \xrightarrow{\text{averaging}} \quad \rho_t = \Phi_t \cdot \rho_0$$

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Morally similar to the *ETH* interpretation which uses the spectral decomposition as canonical unraveling.

An alternative construction of collapse models

Why it is smarter:

1. The constrains are at the master equation level
   → one can start from a well behaved one
2. No need to try to implement the symmetries on a stochastic equation

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Main tool: **non-linear stochastic unraveling**

Given a **non-Markovian** master equation,

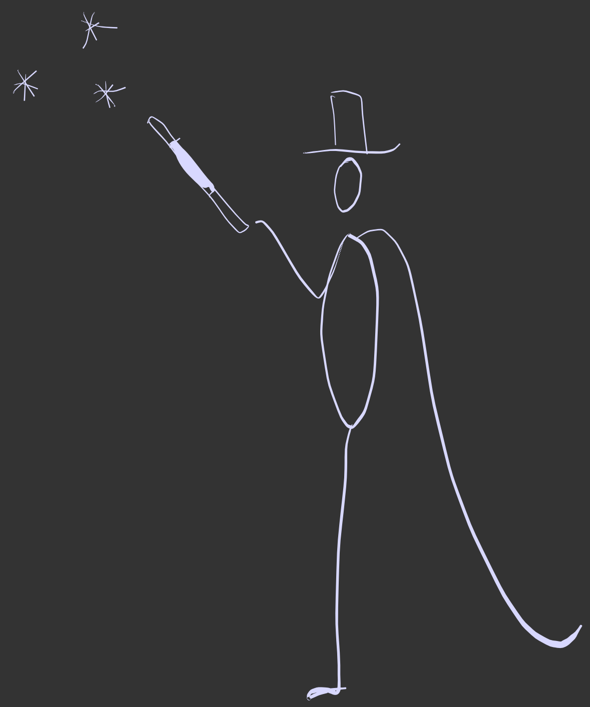
$$\rho_t = \Phi_t \cdot \rho_0$$

that is obtained from tracing out a linearly coupled bosonic bath,

$$H_{\text{int}} = \sum A(\omega) \otimes a^\dagger(\omega) + \text{h.c.}$$

one can construct (infinitely many) stochastic Schrödinger equations for $|\psi\rangle$ unraveling the master equation, i.e. such that:

$$\rho_t = \mathbb{E}\big[|\psi_t\rangle\langle\psi_t|\big]$$



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New question:

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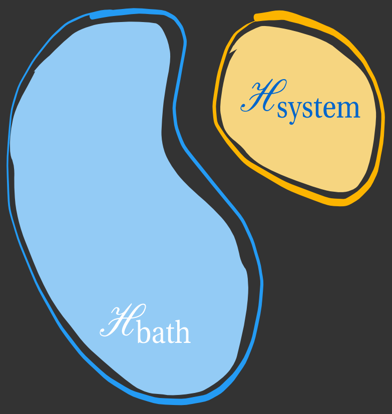
Simple option:

→ A master equation obtained from the **partial trace** of a nice unitary evolution with a bath

$$\mathscr{H}_{\text{tot}} = \mathscr{H}_{\text{system}} \otimes \mathscr{H}_{\text{bath}}$$

$$\partial_t |\Psi\rangle = -i H_{\text{tot}} |\Psi\rangle$$

$$\rho = \text{tr}_B[|\Psi\rangle\langle\Psi|]$$



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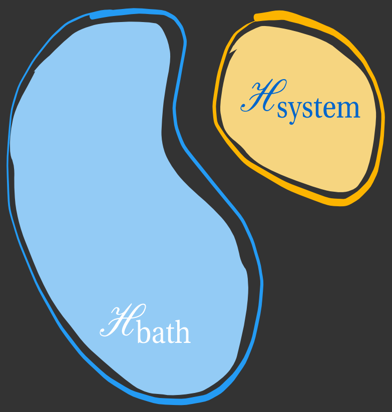

Yet a new question:

What bath to consider?

What bath should we consider?

We want a bath that is:

1) Bosonic
2) Relativistic
3) Linearly coupled to matter (easier to “unravel”)

→ start from an interacting quantum field theory of bosons and fermions (*e.g.* Yukawa theory)

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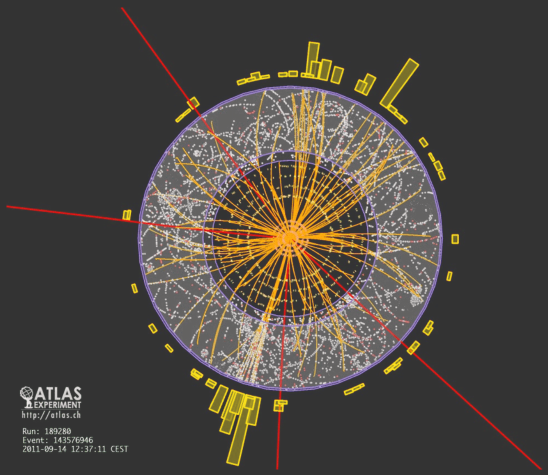
What interacting QFT should one pick?

→ The standard model!



The idea

Tracing out

$\rho_f = \mathrm{tr}_b[|\Psi\rangle\langle\Psi|]$

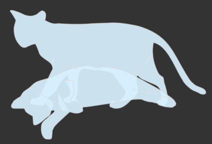
Unraveling

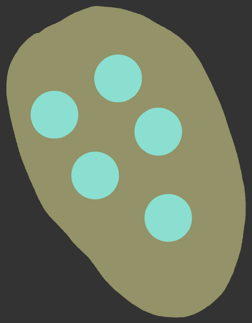
$|\psi_f\rangle$ such that:

$\rho_f = \mathbb{E}[|\psi_f\rangle\langle\psi_f|]$

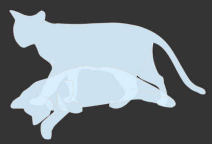
Interacting QFT → **"Open" QFT of fermions** → **Collapse model for fermions**

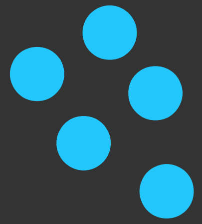
$\partial_t|\Psi\rangle = -iH_{\mathrm{tot}}|\Psi\rangle$



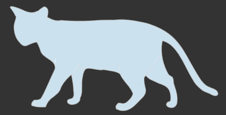


$\rho_f(t) = \Phi_t \cdot \rho_f(0)$





$\partial_t|\psi_f\rangle = -iH_f|\psi_f\rangle + f_\xi(|\psi_f\rangle)$



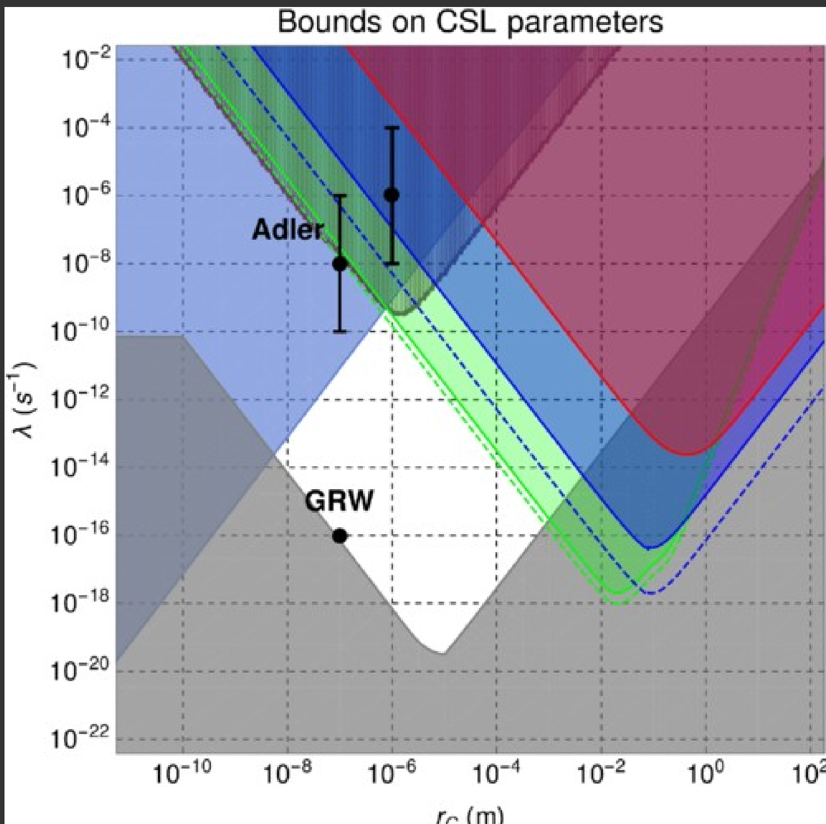
← Empirically equivalent →

Discussion
(accepting the previous construction can be done)

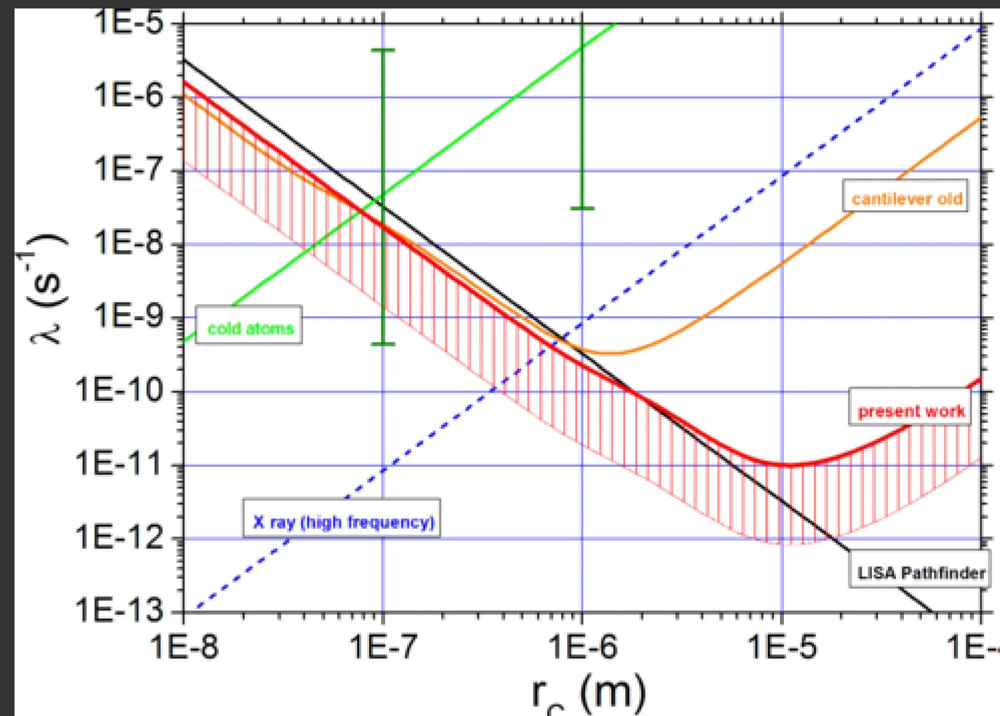
- An extra bath/fundamental force is useless to “explain” collapse (contra Adler, Bassi)
- Divergences are not **in principle** worse than those of standard QFT’s (contra Ghirardi, Pearle, ...)
  → can be perturbatively renormalized
  [actually, the situation is even a little better]
- Collapse models can easily be made **empirically** Lorentz invariant (contra Kochen and Conway)
  [actually, one can even bring the Lorentz invariance at the **ontological** level]
- Relativistic collapse models can have a totally transparent empirical content (contra Pearle / Bedingham / Sudarsky et al.)

Empirical tests: destroying old expectations

Currently, a lot of experimental efforts to probe collapse models.



M. Carlesso, A. Bassi, P. Falferi, and A. Vinante,
Phys. Rev. D **94**, 124036 (2016)



A. Vinante, R. Mezzena, P. Falferi, M. Carlesso, and A. Bassi
Phys. Rev. Lett. **119**, 110401 (2017)

Is it worth the effort?
- Useful to push quantum theory
- But I doubt we find anything like GRW

→ Only way out: forbidding non-Markovianity

New hopes

Collapse model-like reformulations of QFT still have an interest:
→ allow the redefinition of QFTs as statistical field theories

A collapse model reformulation of QFT is ultimately about a random field $\xi(\mathbf{x}, t)$ of local beables [continuous equivalent of flashes] in space-time. The other objects are just tools to compute its probability measure $\mathrm{d}\mu(\xi)$

Such a reformulation is well suited to **fundamental** regularizations preserving the symmetries that are forbidden in orthodox QFT.

$$\frac{1}{1+\mathbf{k}^2} \longrightarrow \frac{1}{1+\mathbf{k}^2+\varepsilon\mathbf{k}^{2N}}$$

A regularized ontology still makes sense, a regularized operational formalism may be meaningless.

Conclusion

1. Collapse models can be *carved into* existing interactions without invoking exotic new ones (similar to ETH).
2. Collapse models can be made relativistic and natural in the context of QFT.
3. This accomplishes the “dynamical reduction program” at the same time as it dissolves it.

Based upon **arXiv:1702.06325**
“*Interacting quantum field theories as relativistic statistical field theories of local beables*”

Bonus: **stochastic unraveling**

<u>The Markovian case:</u>

Consider **Lindblad** equation:

$$\partial_t \rho_t = -i[H,\rho_t] + N\rho N^\dagger - \frac{1}{2}\{N^\dagger N, \rho_t\}$$

Then the stochastic state $|\psi_t\rangle$ obeying the SDE:

$$\mathrm{d}|\psi_t\rangle = \left(-iH\mathrm{d}t + (N - \langle N\rangle)\mathrm{d}W_t - \frac{1}{2}(N-\langle N\rangle)^2\mathrm{d}t\right)|\psi_t\rangle$$

where W_t is a Wiener process, **unravels** the Lindblad equation, i.e.

$$\rho_t = \mathbb{E}[|\psi_t\rangle\langle\psi_t|]$$

The unraveling is **<u>not</u>** unique, but this one is nice:

- It corresponds to the continuous measurement of N
- It is the “maximally collapsing” one with Gaussian noise

The white noise is reminiscent of the Markovian character of the master equation. Going to colored noise will allow to unravel a class of non-Markovian master equations.

The non-Markovian case:

First construct the simplest non-Markovian extension of the **Lindblad** equation:

Consider a system linearly coupled to a bosonic bath:

$$H_{\text{int}} = \hat{A}^i \otimes \hat{\phi}_i$$
$$= \hat{A}^i \otimes \int \mathrm{d}k \, f_i^k a_k^\dagger + f_i^{k*} a_k$$

Start from a product state $\rho_{\text{tot}} = \rho_s \otimes |0\rangle\langle 0|$

Consider the reduced density matrix of the system: $\rho_s = \mathrm{tr}_b[\rho_{\text{tot}}]$

$$\rho_s(t) = \mathcal{T} \exp\left\{ \int_0^t \int_0^t \mathrm{d}u \mathrm{d}v \, D_{ij}(u,v) \Big(A_i^L(u) A_j^R(v) - \theta_{u,v} A_i^L(u) A_j^L(v) - \theta_{v,u} A_j^R(u) A_i^R(v) \Big) \right\} \cdot \rho_s(0)$$

with $D_{ij}(u,v) = \mathrm{tr}\left[\hat{\phi}(u) \hat{\phi}(v) \rho_b(0) \right]$

[Just an operator rewriting of the Feynman-Vernon influence functional]

Gives back the Lindblad equation for $D_{ij}(u,v) \to C_{ij} \delta(u-v)$

Diósi, L., & Ferialdi, L. (2014). Phys. Rev. Lett., **113**(20), 200403.

We want to unravel the master equation:

$$\rho_s(t) = \mathscr{T} \exp\left\{\int_0^t \int_0^t \mathrm{d}u\mathrm{d}v\, D_{ij}(u,v) \Big(A_i^L(u) A_j^R(v) - \theta_{u,v} A_i^L(u) A_j^L(v) - \theta_{v,u} A_j^R(u) A_i^R(v) \Big) \right\} \cdot \rho_s(0)$$

Looks very much like the generating functional of a complex Gaussian noise with D as two-point function. With trial and error, one finds:

$$|\psi_\xi(t)\rangle = \mathscr{T} \exp\left\{ -i \int_0^t \mathrm{d}s \hat{A}^k(s) \xi_k(s) - \int_0^t \int_0^t \mathrm{d}\tau\, \mathrm{d}s \theta_{\tau s} [D-S]_{ij}(\tau,s) \hat{A}^i(\tau) \hat{A}^j(s) \right\} |\psi_0\rangle$$

or equivalently in “differential” form:

$$\frac{\mathrm{d}}{\mathrm{d}t} |\psi_\xi(t)\rangle = -i \hat{A}^i(t) \left[\xi_i(t) + \int_0^t \mathrm{d}s\, [D-S]_{ij}(t,s) \frac{\delta}{\delta \xi_j(s)} \right] |\psi_\xi(t)\rangle$$

where ξ is a complex Gaussian field of two point functions:

$$\mathbb{E}_\xi \left[\xi_i(\tau) \xi_j^*(s) \right] = D_{ij}(\tau,s)$$

$$\mathbb{E}_\xi \left[\xi_i(\tau) \xi_j(s) \right] = S_{ij}(\tau,s)$$

So far:
- S is a free parameter
- The norm is not preserved

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$$\frac{\mathrm{d}}{\mathrm{d}t}|\psi_\xi(t)\rangle = -i\hat{A}^i(t)\left[\xi_i(t) + \int_0^t \mathrm{d}s\,[D-S]_{ij}(t,s)\frac{\delta}{\delta\xi_j(s)}\right]|\psi_\xi(t)\rangle$$

where ξ is a complex Gaussian field of two point functions:

$$\mathbb{E}_\xi\left[\xi_i(\tau)\xi_j^*(s)\right] = D_{ij}(\tau,s)$$
$$\mathbb{E}_\xi\left[\xi_i(\tau)\xi_j(s)\right] = S_{ij}(\tau,s)$$

To conserve the norm, one normalizes the state **and** changes field probability measure.

$$|\widetilde{\psi}_\xi(t)\rangle = \frac{|\psi_\xi(t)\rangle}{\sqrt{\langle\psi_\xi(t)|\psi_\xi(t)\rangle}}$$
$$\mathrm{d}\mu_t(\xi) = \langle\psi_\xi(t)|\psi_\xi(t)\rangle\,\mathrm{d}\mu_\circ(\xi)$$

This choice makes the state unravel the master equation for a single time t. To get all times one needs to make **a continuous change of field variable** such that:

$$\forall f, \qquad \mathbb{E}_\xi^t[f(\xi)] = \mathbb{E}_\xi\left[f\left(\xi^{[t]}(\xi)\right)\right]$$

After painful computations one can show that the transformed field variable verifies:

$$\xi_k^{[t]}(u) = \xi_k(u) + i\int_0^t \mathrm{d}s\,D_{k\ell}(u,s)\langle A^\ell(s)\rangle_s$$

In the end, the normalized state for the transformed field variable unravels the master equation!

L. Diósi, N. Gisin, and W. T. Strunz (1998) Phys. Rev. A **58**, 1699
Diósi, L., & Ferialdi, L. (2014). Phys. Rev. Lett., **113**(20), 200403.
A Tilloy (2017), Quantum **1**, 29

Super bonus: **local beables**

Consider the field ξ transformed for infinite time. For a quantum field theory it has the following properties:

- The definition of its probability measure is foliation independent
- Its probability measure fully Lorentz invariant in the vacuum
- It behaves like the GRW flashes (gives well defined macroscopic objects via coarse graining)

Promising for a stochastic field theory **redefinition** of QFT?