Continuous measurement in the large γ limit

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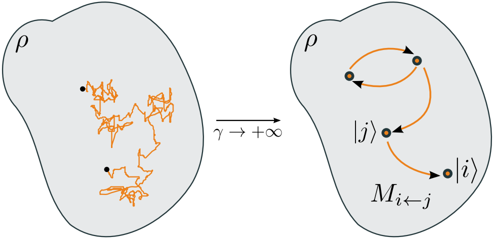

Theory seminar
Griffith University
March 26th, 2018

Alexander von Humboldt
Stiftung/Foundation
MPQ

Genesis

Work done in Paris at ENS with

- Denis Bernard (ENS, Paris)
- Michel Bauer (CEA, Saclay)

Corresponds to my PhD **thesis** and:

1. *Computing the rate of measurement induced quantum jumps*
   `arXiv:1410.7231`
2. *Spikes in quantum trajectories*
   `arXiv:1510.01232`
3. *Zoomin in on quantum trajectories*
   `arXiv:1512.02861`

Objective

Understand dynamics of the type:

$$\partial_t \rho_t = \mathcal{L}(\rho_t) + \gamma\, \mathcal{M}(\rho_t)$$

where:

- $\mathcal{L}$ is the **Liouvillian** in absence of measurement
- $\mathcal{M}$ encodes a **continuous measurement** process

when $\gamma \to +\infty$

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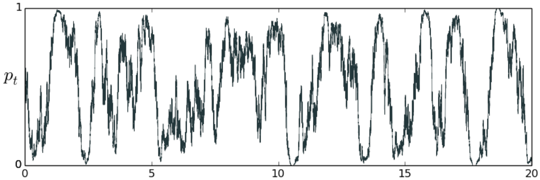
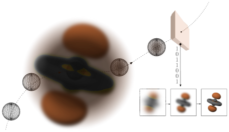
Analysis at the **trajectory** level:

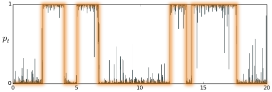


Outline

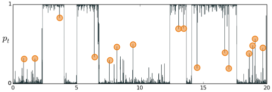
1. Introduction: continuous measurement



2. Jumps

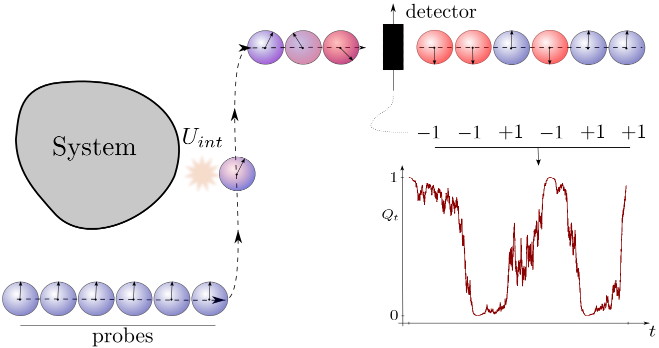


3. Spikes



4. Discussion

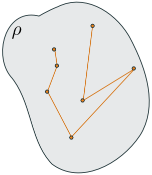
Continuous measurement



Repeated interactions

Discrete quantum trajectories

A sequence of $|\psi_n\rangle$ or ρ_n (random) and the corresponding measurement results $\delta_n = \pm 1$.



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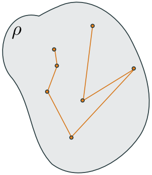


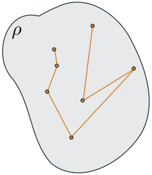
- Make the interaction between system and probe smoother

$$U_{\text{int}} = \mathbb{1} + i\varepsilon\, \mathcal{O}_{\text{sys}} \otimes K_{\text{probe}}$$

- Increase the frequency at which probes are sent:

$$\tau \propto \varepsilon$$

Repeated interactions



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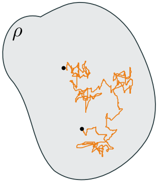
Continuous quantum trajectories

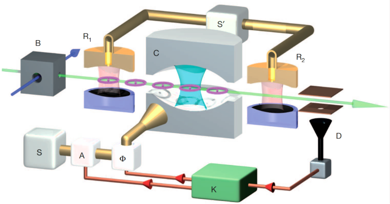
A continuous map $|\psi_t\rangle$ or ρ_t (random) and the corresponding continuous measurement signal $y_t \propto \sqrt{\varepsilon}\sum_k \delta_k$



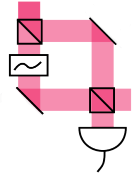
⚠Essentially a central limit theorem result ⚠

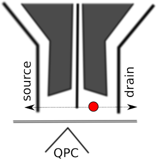
In practice

- Discrete situations “a la Haroche”, with **actual** repeated interactions



- Almost “true” continuous measurement settings (homodyne detection in quantum optics, quantum point contacts for quantum dots)





Result

Stochastic Master Equation (~ 1987)

Density matrix:

$$d\rho_t = \mathcal{L}(\rho_t)\, dt + \gamma\, \mathcal{D}[\mathcal{O}](\rho_t)\, dt + \sqrt{\gamma}\, \mathcal{H}[\mathcal{O}](\rho_t)\, dW_t$$

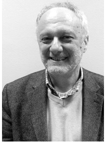
Signal:

$$dy_t = \sqrt{\gamma}\, \mathrm{tr}\left[\left(\mathcal{O} + \mathcal{O}^\dagger\right) \rho_t\right] dt + dW_t$$

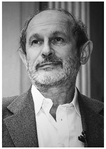
with:

- $\mathcal{D}[\mathcal{O}](\rho) = \mathcal{O}\rho\mathcal{O}^\dagger - \frac{1}{2}\left(\mathcal{O}^\dagger\mathcal{O}\rho + \rho\mathcal{O}^\dagger\mathcal{O}\right)$
- $\mathcal{H}[\mathcal{O}](\rho) = \mathcal{O}\rho + \rho\mathcal{O}^\dagger - \mathrm{tr}\left[\left(\mathcal{O} + \mathcal{O}^\dagger\right)\rho\right]\rho$
- $\frac{dW_t}{dt}$ "white noise"

V. Belavkin



A. Barchielli



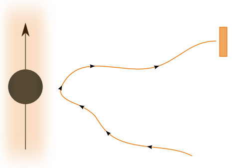
L. Diósi

Example 0

Situation considered

Pure continuous measurement of a qubit:

- Qubit $\Rightarrow \mathscr{H} = \mathbb{C}^2$
- Hence $\rho_t = \begin{pmatrix} p_t & u_t \\ u_t^* & 1 - p_t \end{pmatrix}$
- Continuous energy measurement: $\mathcal{O} = \sigma_z \propto H$



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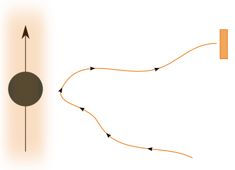

Starting point:

$$\mathrm{d}\rho_t = \gamma\, \mathcal{D}[\mathcal{O}](\rho_t)\, \mathrm{d}t + \sqrt{\gamma}\, \mathcal{H}[\mathcal{O}](\rho_t)\, \mathrm{d}W_t$$

$\Longrightarrow$ **Equation for the probability**:

$$\mathrm{d}p_t = \sqrt{\gamma}\, p_t(1 - p_t)\, \mathrm{d}W_t$$

$\Longrightarrow$ **Equation for the phase**:

$$\mathrm{d}u_t = -\frac{\gamma}{8} u_t\, \mathrm{d}t + \frac{\sqrt{\gamma}}{2}(2p_t - 1)\, \mathrm{d}W_t$$

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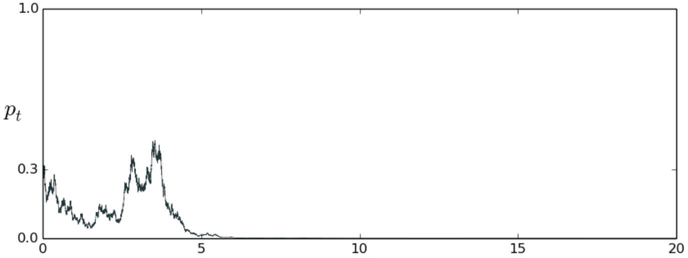

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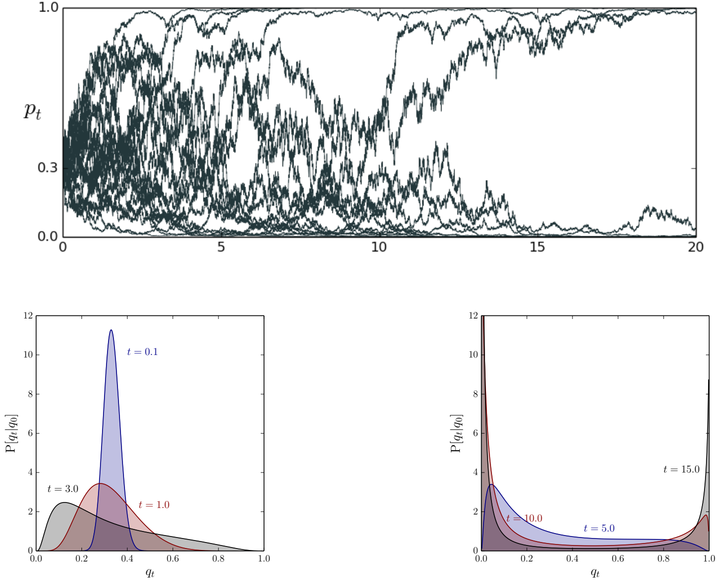
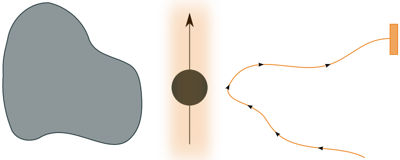


Example 1

Qubit coupled to a thermal bath

System considered

- Qubit $\mathscr{H} = \mathbb{C}^2$
- Continuous measurement of $\mathcal{O} \propto H \propto \sigma_z$
- Markovian thermal bath
- $\rho_t = \begin{pmatrix} p_t & u_t \\ u_t^* & 1 - p_t \end{pmatrix}$



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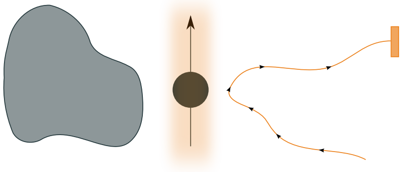
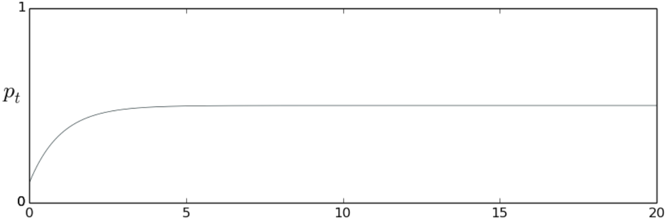


Autonomous stochastic master equation for p_t:

$$\mathrm{d}p_t = \underbrace{\lambda(p_{\mathrm{eq}} - p_t)\,\mathrm{d}t}_{\text{thermal relaxation}} + \underbrace{\sqrt{\gamma}\,p_t(1 - p_t)\,\mathrm{d}W_t}_{\text{continuous measurement}}$$

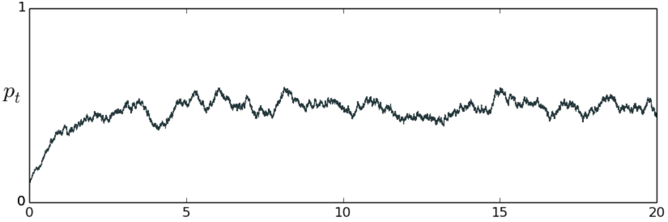
Example 1

No measurement, $\gamma = 0\,\lambda$



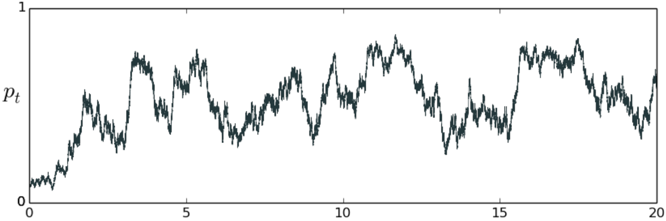
Example 1

Weak measurement, $\gamma = 0.1\,\lambda$



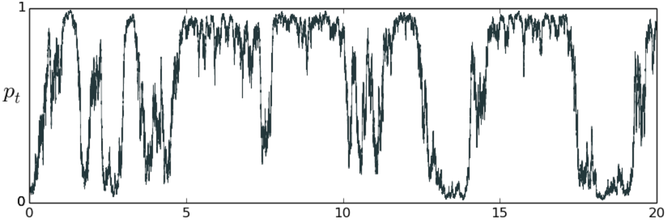
Example 1

Decent measurement, $\gamma = \lambda$



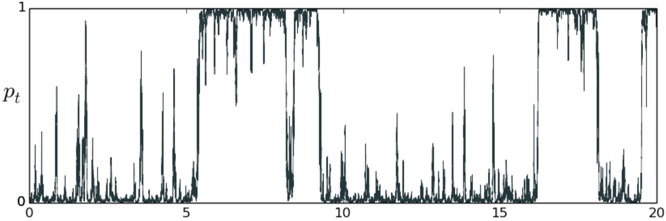
Example 1

Getting strong measurement, $\gamma = 10\,\lambda$



Example 1

Pretty strong measurement, $\gamma = 100\,\lambda$



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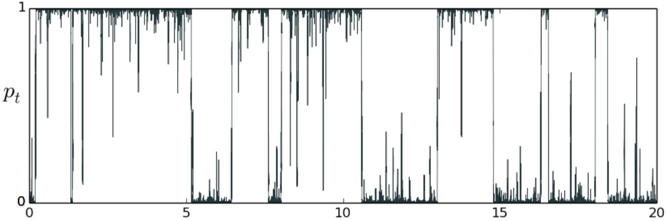
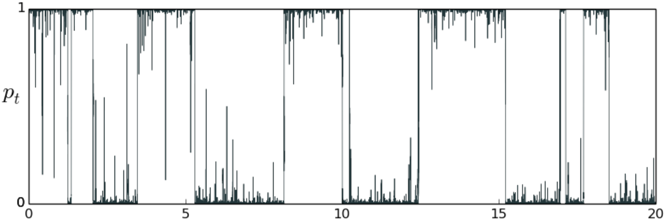


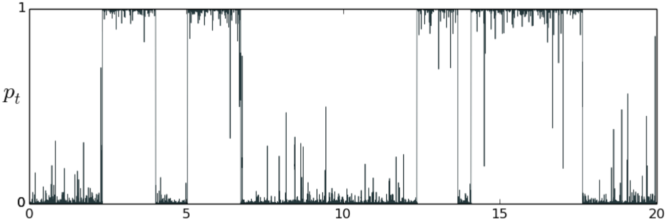
Example 1

Very strong measurement, $\gamma = 10^4\,\lambda$



Example 1

Über strong measurement, $\gamma = 10^5\,\lambda$



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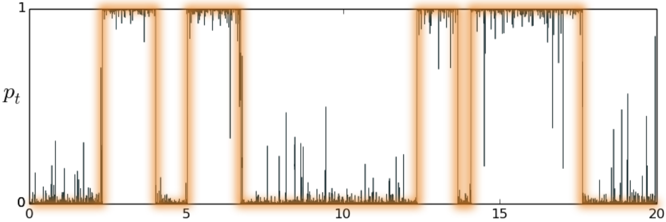
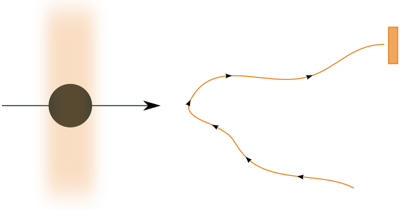


Example 2

Measurement non-commuting with the evolution

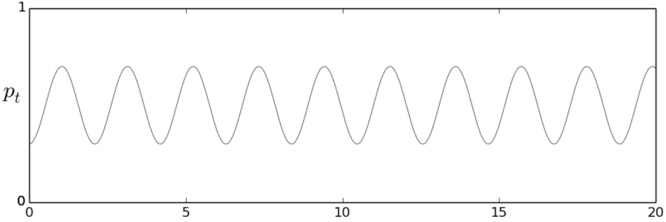
System considered

- Continuous measurement of $\mathcal{O} \propto \sigma_z \perp H$
- Closed system
- $\rho_t = \begin{pmatrix} p_t & u_t \\ u_t^* & 1 - p_t \end{pmatrix}$



Example 2

No measurement, $\gamma = 0\,\omega$



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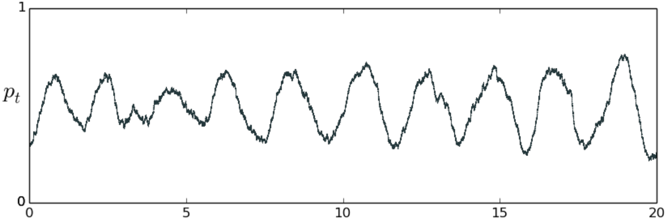
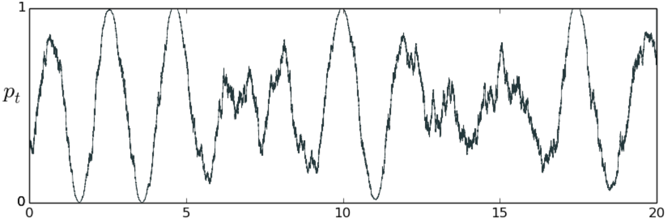


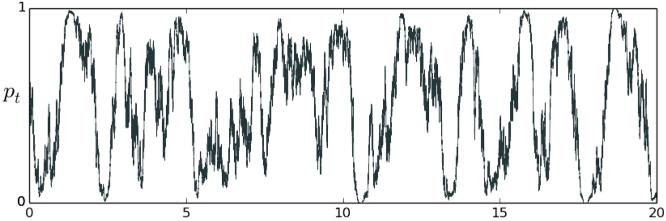
Example 2

Decent measurement, $\gamma = \omega$



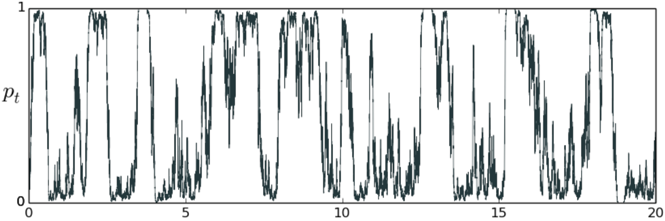
Example 2

Getting strong measurement, $\gamma = 10\,\omega$



Example 2

Pretty strong measurement, $\gamma = 30\,\omega$



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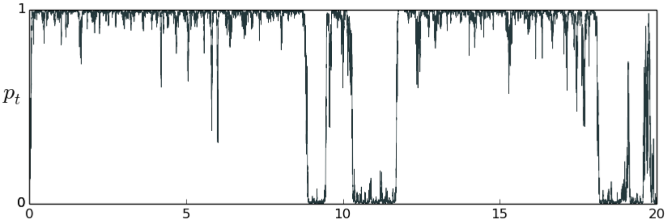
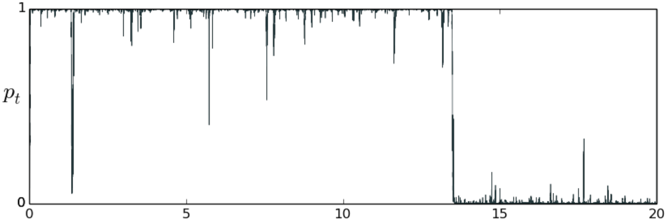
Strong measurement, $\gamma = 100\,\omega$



Example 2

Very strong measurement, $\gamma = 300\,\omega$



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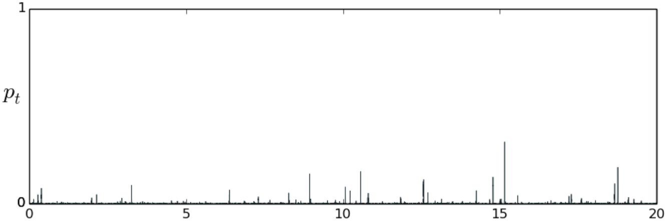

Theorem: jumps

Consider $d\rho = \mathcal{L}(\rho)\,dt + \gamma\mathcal{D}[\mathcal{O}](\rho)\,dt + \sqrt{\gamma}\,\mathcal{H}[\mathcal{O}](\rho)\,dW_t$

1. Markovian evolution $\mathcal{L}(\rho_t) = L(\rho_t) - i\,[H, \rho_t]$
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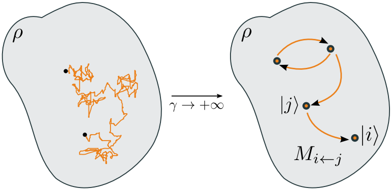
Quantum jumps

When $\gamma \to +\infty$, ρ_t converges to a **Markov chain** with transition matrix M:

$$M_{i\leftarrow j} = \overbrace{L_{jj}^{ii}}^{\text{“incoherent” contribution}} + \underbrace{\frac{1}{4\gamma}\left|\frac{H_{ij}}{\lambda_i - \lambda_j}\right|^2}_{\text{“incoherent” contribution}}$$



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Extensions

- Several commuting observables $\mathcal{O}_\ell$

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Extensions

- Several commuting observables $\mathcal{O}_\ell$
- Repeated imperfect measurements instead of continuous

Jumps: proof

Standard small noise expansion techniques are useless in this context

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Idea of the proof
Perturbation theory at the level of the Fokker-Planck equation for ρ_t:

$$\partial_t \mathcal{P}(\rho) = \mathfrak{D}(\rho)$$

where $\mathfrak{D}$ is a differential operator

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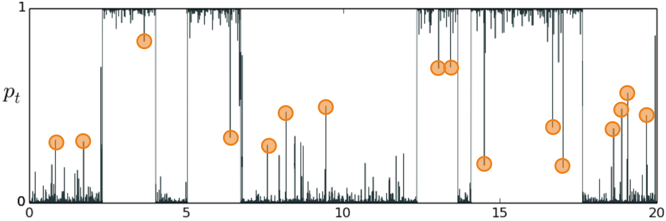
where $\mathfrak{D}$ is a differential operator

Write $\mathfrak{D} = \gamma \mathfrak{D}_1 + \mathfrak{D}_0$, hence $\mathcal{P}(\rho) = \exp\left(t\gamma \mathfrak{D}_1 + t\mathfrak{D}_0\right)$

- To zeroth order, $\mathcal{P}(\rho) = \exp(t\gamma D_1)$, $\implies$ converges exponentially fast to the kernel of $\mathfrak{D}_1$, i.e. Dirac around **pointer states**
- To next order, $\exp t\mathfrak{D}_0$ gives the transition rates

Spikes



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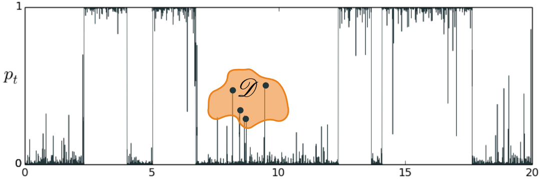
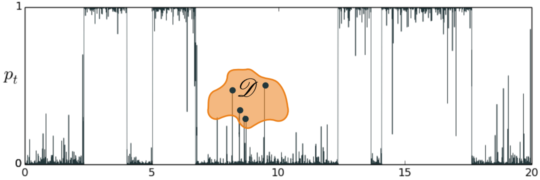


Theorem: spikes



Spike statistics

The number of spikes **starting from** 0 and ending in the domain $\mathscr{D}$ of the plane (t, p) is a Poisson process of intensity $\mu(D)$:

$$\mu = \int_{\mathscr{D}} d\nu \quad \text{with} \quad d\nu = \frac{\lambda}{p^2} dp\, dt$$

Spikes: idea of the proof

Quickest way: do a ρ dependent time rescaling – arXiv:1512.02861

$$p_t^2(1-p_t)^2\,dt = d\tau$$

p_τ has a well defined limit when $\gamma \to +\infty$:

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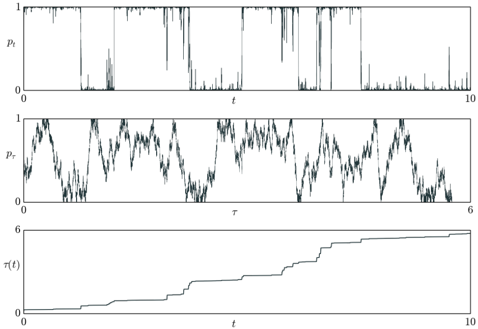

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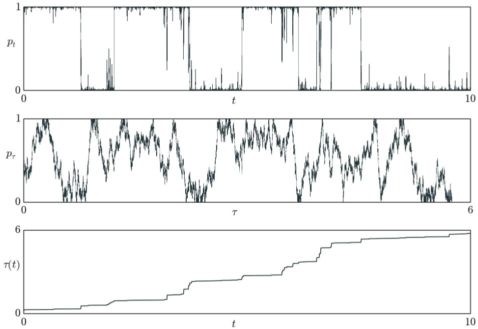
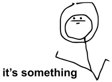
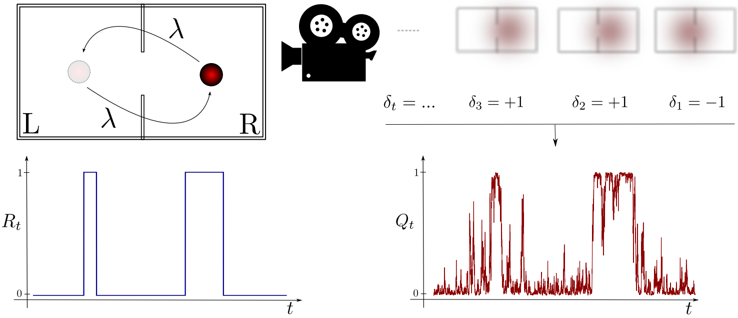


Works only for qubits...



Are spikes real?

Introduce a classical hidden Markov model:



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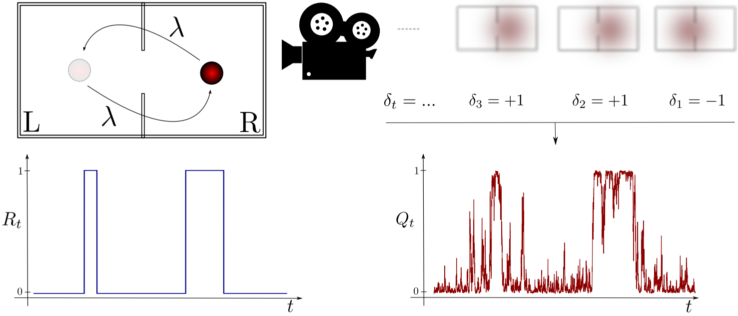
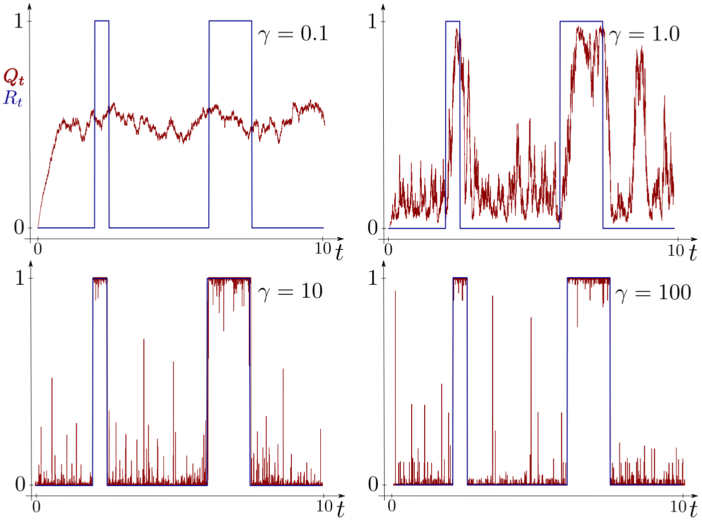


Yields the same filtering equation as for thermal jumps:

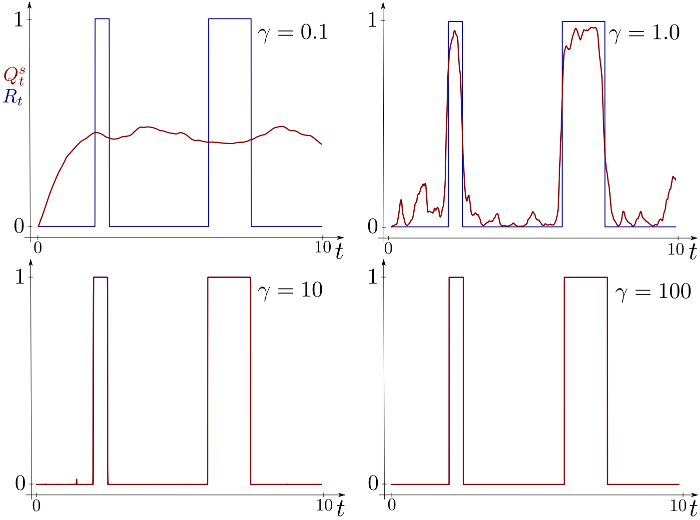
$$\mathrm{d}Q_t = \lambda(Q_{\mathrm{eq}} - Q_t)\,\mathrm{d}t + \sqrt{\gamma}\,Q_t(1 - Q_t)\,\mathrm{d}W_t$$

Are spikes real?



Are spikes real?

With (classical) smoothing, i.e. a posteriori estimation:



Spikes: summary

"Ontologically"
Spikes are not an exclusively quantum phenomenon but can exist in genuinely quantum settings:

1. Spikes with classical Hidden Markov Models
2. Spikes with states pure at all time

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In practice

- Spikes can make control difficult
- Spikes are not (necessarily) coming from classical errors

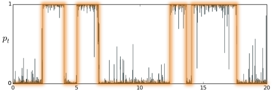
A few (possibly difficult) open questions

1. Continuous measurement of observables with continuous spectra like $\hat{X}$
2. Continuous measurement in the many-body context – phase transition?
3. Spikes in $d \geqslant 3$
4. Similar strong noise limits in other contexts:
   scalar turbulence? avalanches? finance? e.g. Henkel `arXiv:1609.05286`

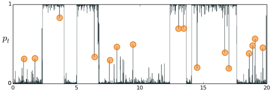
General summary

Strong continuous measurement yields:

1. Jumps



2. Spikes



Jumps

1. Can be fully characterized
2. Are Zeno frozen if coherent, not frozen if incoherent
3. Quantitatively different from projective measurement

Spikes

1. Can be characterized for qubits
2. Are power law distributed, with infinitely many small ones
3. Are not exclusively quantum but sometimes purely quantum