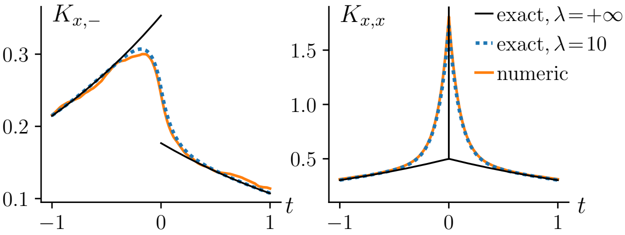
Signal correlation functions for parameter estimation

Antoine Tilloy
Max Planck Institute of Quantum Optics, Garching, Germany



Observability and estimation in quantum dynamics
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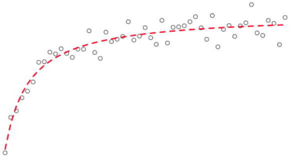
MPQ

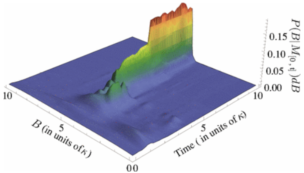
Introduction

There are often two ways to **estimate** things:

- The **dumb** way: compute theory prediction and fit on simple observables.
- The **smart** way: reconstruct the optimal Bayesian estimate of the thing from the dataset.



“Dumb” way



“Smart” way

Figure: Chase & Geremia 2009, *Phys. Rev.* A **79**, 022314

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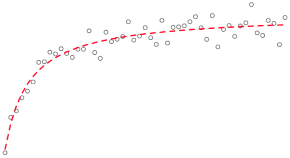

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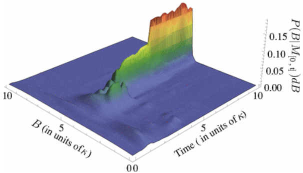


"Smart" way

Objective: discuss both approaches in the continuous measurement context

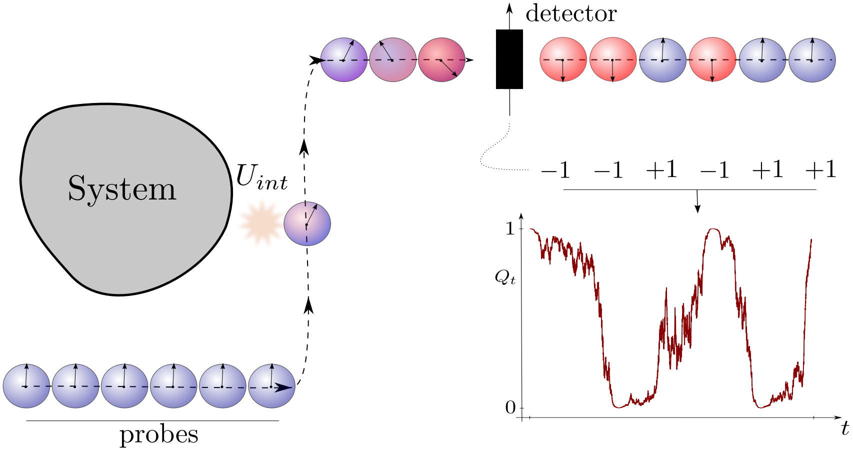
Small novelty: A theoretical formula for the N-point functions makes the **dumb** way quite general

Figure: Chase & Geremia 2009, *Phys. Rev.* A **79**, 022314

Outline

1. Continuous measurements
2. Smart parameter estimation
3. Dumb parameter estimation
4. Computing correlation function
5. Including imperfections

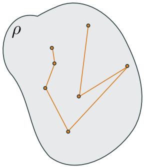
Continuous measurement



Repeated interactions

Discrete quantum trajectories

A sequence of $|\psi_n\rangle$ or ρ_n (random) and the corresponding measurement results $\delta_n = \pm 1$.



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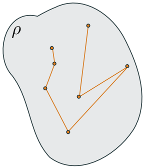

- Make the interaction between system and probe smoother

$$U_{\text{int}} = \mathbb{1} + i\sqrt{\varepsilon}\,\mathcal{O}_{\text{sys}} \otimes K_{\text{probe}}$$

- Increase the frequency at which probes are sent:

$$\tau \propto \varepsilon$$

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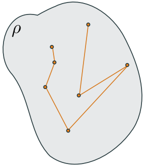

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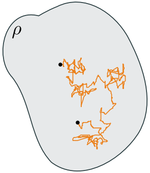
Continuous quantum trajectories

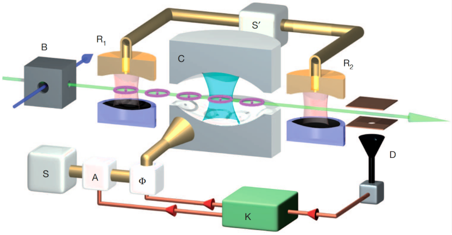
A continuous map $|\psi_t\rangle$ or ρ_t (random) and the corresponding continuous measurement signal $r_t \propto \sqrt{\varepsilon}\sum_k \delta_k$



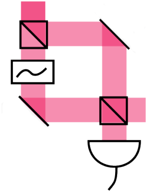
⚠Essentially a central limit theorem result ⚠

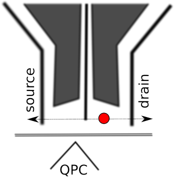
In practice

- Discrete situations "*LKB* style", with **actual** repeated interactions



- Almost "true" continuous measurement settings (homodyne detection in quantum optics, quantum point contacts for quantum dots)





Result

Stochastic Master Equation **($\sim$ 1987)**

Density matrix:

$$d\rho_t = \mathcal{L}(\rho_t)\,dt + \gamma\,\mathcal{D}[\mathcal{O}](\rho_t)\,dt + \sqrt{\gamma\eta}\,\mathcal{H}[\mathcal{O}](\rho_t)\,dW_t$$

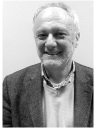
Signal:

$$I_t = \frac{dr_t}{dt} \text{ with } dr_t = \sqrt{\gamma}\,\mathrm{tr}\left[(\mathcal{O} + \mathcal{O}^\dagger)\,\rho_t\right]\,dt + dW_t$$

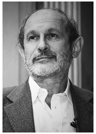
with:

- $\mathcal{D}[\mathcal{O}](\rho) = \mathcal{O}\rho\mathcal{O}^\dagger - \frac{1}{2}\left(\mathcal{O}^\dagger\mathcal{O}\rho + \rho\mathcal{O}^\dagger\mathcal{O}\right)$
- $\mathcal{H}[\mathcal{O}](\rho) = \mathcal{O}\rho + \rho\mathcal{O}^\dagger - \mathrm{tr}\left[(\mathcal{O} + \mathcal{O}^\dagger)\,\rho\right]\rho$
- $\frac{dW_t}{dt}$ "white noise"

V. Belavkin



A. Barchielli



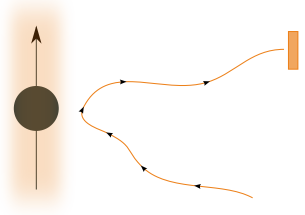
L. Diósi

Example 0

Situation considered

Pure continuous measurement of a qubit:

- Qubit $\Rightarrow \mathscr{H} = \mathbb{C}^2$
- Hence $\rho_t = \begin{pmatrix} p_t & u_t \\ u_t^* & 1 - p_t \end{pmatrix}$
- Continuous energy measurement: $\mathcal{O} = \sigma_z \propto H$



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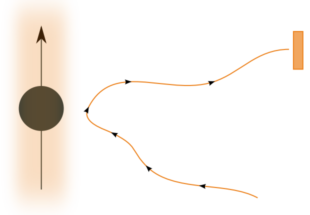

Starting point:

$$d\rho_t = \gamma\, \mathcal{D}[\mathcal{O}](\rho_t)\, dt + \sqrt{\gamma}\, \mathcal{H}[\mathcal{O}](\rho_t)\, dW_t$$

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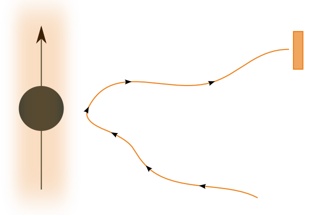

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$\Longrightarrow$ **Equation for the probability**:

$$dp_t = \sqrt{\gamma}\, p_t(1 - p_t)\, dW_t$$

$\Longrightarrow$ **Equation for the phase**:

$$du_t = -\frac{\gamma}{8} u_t\, dt + \frac{\sqrt{\gamma}}{2}(2p_t - 1)\, dW_t$$

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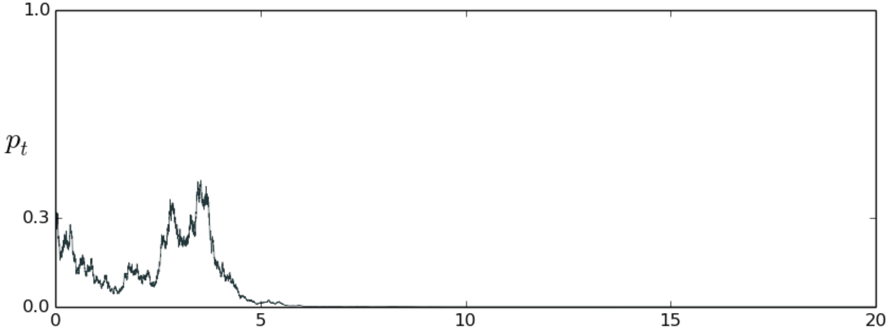
Pure continuous measurement of a qubit $\mathrm{d}p_t = \sqrt{\gamma}\, p_t(1 - p_t)\,\mathrm{d}W_t$



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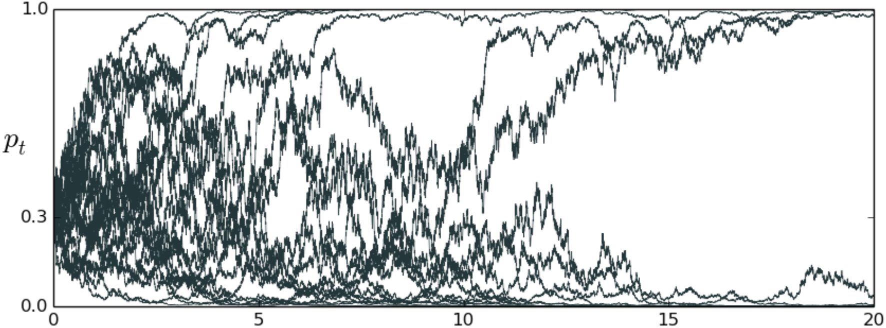

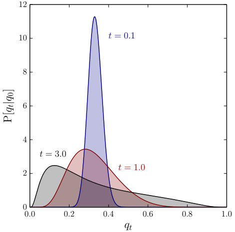

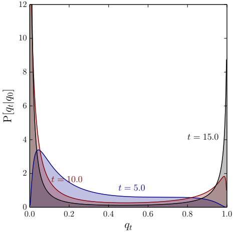

Smart parameter estimation

We assume there are unknown parameters in the evolution, e.g. ρ evolves with:

$$d\rho_t = -i[H_\xi, \rho_t]\,dt + \underbrace{\gamma\mathcal{D}[\mathcal{O}](\rho_t)\,dt + \sqrt{\gamma\eta}\mathcal{H}[\mathcal{O}](\rho_t)\,dW_t}_{\text{continuous measurement}}$$

with ξ unknown.

See *e.g.*: Gambetta & Wiseman 2001,*Phys. Rev. A* **64**, 042105 — Chase & Geremia 2009, *Phys. Rev.* A **79**, 022314 — Negretti & Mølmer 2013, *New J. Phys.* **15**, 125002 — Six *et al.* 2015, *54th IEEE CDC*, pp. 7742-7748

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Question

How to estimate ξ from a continuous measurement record I_t ?

Idea

Turn parameter estimation back into a filtering problem.

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Turn parameter estimation back into a filtering problem.

Imagine the unknown parameter is an **internal** degree of freedom:

1. Define $\mathscr{H} = \mathscr{H}_s \otimes \mathscr{H}_{aux}$ with $\mathscr{H}_{aux} = \text{span}(|\xi_1\rangle, \cdots, |\xi_N\rangle)$
2. Define $\rho_0 = \sum_{k=1}^N \rho_s^k \otimes |\xi_k\rangle\langle\xi_k|$ where $\rho_s^k = \pi_k\, \rho_s(0)$ where π_k prior for ξ_k
3. Define $H = \sum_{k=1}^N H_{\xi_k} \otimes |\xi_k\rangle\langle\xi_k|$, and $\mathcal{O} \to \mathcal{O} \otimes \mathbb{1}_{aux}$
4. Solve the SME: $d\rho_t = -i[H, \rho_t]\,dt + \gamma\mathcal{D}[\mathcal{O}](\rho_t)\,dt + \sqrt{\gamma\eta}\mathcal{H}[\mathcal{O}](\rho_t)\,dW_t$

Diagonality in the auxiliary basis is preserved and generically $\pi_k(t) \underset{t\to+\infty}{\sim} \delta_{k,k_{real}}$

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Smart parameter estimation: intuition

Why does it work?

Convergence? $\rightarrow$ Collapse in the space of parameters

Continuous measurement purifies the state enlarged with its "unknown parameter" degree of freedom while preserving the block diagonal structure.

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Convergence? → Collapse in the space of parameters

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Accuracy? → Hidden Variable

Because we are diagonal in the basis of unknown parameters, the model is mathematically equivalent to a classical filter with an unknown yet predetermined hidden variable ξ.

Easy to extend to a fluctuating unknown parameter $\xi(t)$.

Smart parameter estimation: first subtlety

Discretization of the unkown parameter values $\xi \to \xi_k$
The auxiliary Hilbert space $\mathscr{H}_{\text{aux}}$ needs to be **large** for high accuracy

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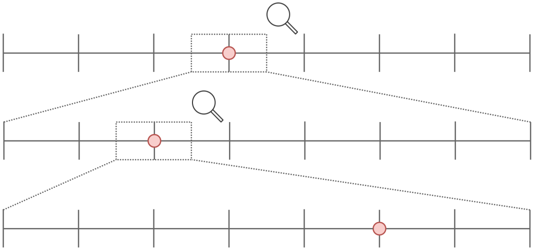
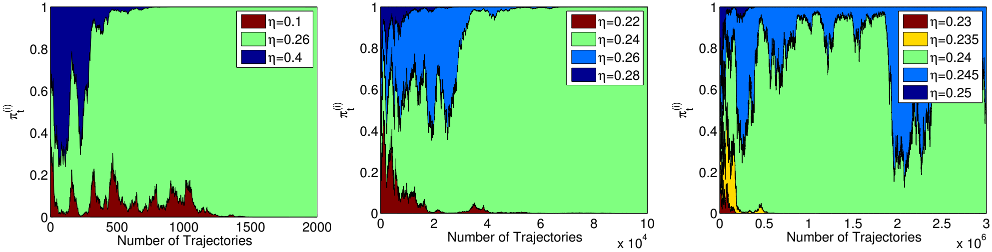
Resampling
Keep $\mathscr{H}_{\text{aux}}$ small to find a crude approximation and then refine around this value. Iterate.



Smart parameter estimation: example

Estimation of the efficiency η of the heterodyne detection of a fluorescence:



Six *et al.* 2015, *54th IEEE Conference on Decision and Control (CDC)*, Osaka, pp. 7742-7748

Smart parameter estimation: difficulties

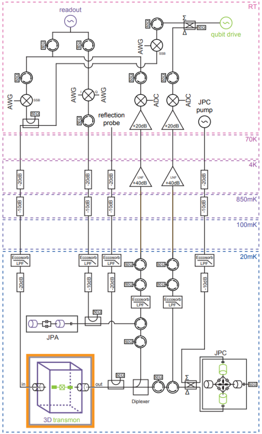
a) Main practical difficulty: $\rho_t = f(\rho_0, I)$ where f is a complicated non-linear function of the measured signal.
b) Further, amplification typically filters the signal:

$$\underbrace{\widetilde{I}_t}_{\text{measured signal}} = \int_{-\infty}^{t} \mathrm{d}u \, K(t-u) \underbrace{I_u}_{\text{true signal}}$$

Solution:

1. **Discretize** at a high frequency
2. **Deconvolute** the filter of the detection chain
3. **Numerically integrate** the discretized SME

But will become untractable for large quantum systems because requires reconstructing ρ.



detection chain

Figure: *Supplementary Material* of Ficheux *et al.* 2017, `arXiv:1711.01208`
Smart integration scheme: Rouchon & Ralph 2015, *Phys. Rev. A* **91**, 012118

Dumb parameter estimation

Smart parameter estimation is **mathematically optimal**, but sometimes overkill or hard to implement.

Smart parameter estimation

Reconstruct a non-trivial function of the signal *e.g.* ρ_t to obtain directly the probability π of the unknown parameter ξ.

Dumb parameter estimation

Compute a trivial function of the signal *e.g.* $\mathbb{E}[I_{t_1} I_{t_2}]$ and use the **theory** to relate it to system parameters ξ. **Fit** theory on experiment.

See *e.g.*: Jordan & Büttiker 2005, *Phys. Rev. Lett.* **95**, 220401 — Atalaya *et al.* 2017, *Phys. Rev. A* **95**, 032317 — Atalaya *et al.* 2017, `arXiv:1702.08077`

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Candidates for "dumb" parameter estimation

→ the signal N-point correlation functions:

$$K_{t_1,\cdots,t_N} = \mathbb{E}\left[I_{t_1} \cdots I_{t_N}\right]$$

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Candidates for "dumb" parameter estimation

$\rightarrow$ the signal N-point correlation functions:

$$K_{t_1,\cdots,t_N} = \mathbb{E}\left[I_{t_1} \cdots I_{t_N}\right]$$

Advantages

1. Requires only analog manipulations of the signal (delay lines & multipliers)
2. Can include all the imperfections in the theoretical fit.

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Computing correlation functions

Setup: Continuous measurement of several operators

$$\mathrm{d}\rho = \mathcal{L}(\rho)\mathrm{d}t + \sum_{k=1}^{n} \mathcal{D}[c_k](\rho)\,\mathrm{d}t + \sqrt{\eta_k}\,\mathcal{H}[c_k](\rho)\,\mathrm{d}W_k,$$

with signal:

$$I_k = \frac{\mathrm{d}r_k}{\mathrm{d}t} \text{ with } \mathrm{d}r_k = \frac{1}{2}\mathrm{tr}[(c_k + c_k^\dagger)\rho]\,\mathrm{d}t + \frac{1}{2\sqrt{\eta_k}}\mathrm{d}W_k.$$

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Objective

Compute N-point functions:

$$K_{\ell_1\ell_2\cdots\ell_N}(t_1, t_2, \cdots, t_N) := \mathbb{E}\left[I_{\ell_1}(t_1) I_{\ell_2}(t_2) \cdots I_{\ell_N}(t_N)\right],$$

as a function of the system dynamics $\mathcal{L}$, measured operators $\{c_k\}_{k=1}^n$, and efficiencies $\{\eta_k\}_{k=1}^n$.

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Many ways:

- Quantum noise / continuous matrix product states
- Discretization

But I take the (possibly pedestrian) **purely stochastic route**.

Computing correlation functions

Idea:

1. Introduce a generating functional $\mathcal{Z}_{\mathbf{j}}(T)$,

$$\mathcal{Z}_{\mathbf{j}} := \mathbb{E}\left[\exp\left(\sum_{k=1}^{n}\int_0^T j_k(u)\, I_k(u)\,\mathrm{d}u\right)\right]$$

2. Solve a differential equation for $\bar{\rho}_{\mathbf{j}}(T)$ such that $\mathrm{tr}[\bar{\rho}_{\mathbf{j}}(T)] = \mathcal{Z}_{\mathbf{j}}(t)$,
3. Carry the functional differentiation explicitly

$$K_{\ell_1\cdots\ell_N}(t_1,\cdots,t_N) = \frac{\delta}{\delta j_{\ell_1}(t_1)}\cdots\frac{\delta}{\delta j_{\ell_N}(t_N)}\mathcal{Z}_{\mathbf{j}}\Big|_{\mathbf{j}=0}$$

Computing correlation functions

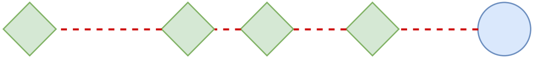
Correlation function – without equal point distribution

For $t_1 < t_2 < \cdots < t_N$:

$$K_{\ell_1\ell_2\cdots\ell_N}(t_1, t_2, \cdots, t_N) = \frac{1}{2^N} \times \mathrm{tr}\left[c^+_{\ell_N}\Phi_{t_N - t_{N-1}} c^+_{\ell_{N-1}} \cdots \Phi_{t_2-t_1} c^+_{\ell_1}\Phi_{t_1} \cdot \rho(0)\right],$$

where $\Phi_t = \exp\{t\,(\mathcal{L} + \sum_{k=1}^{n} \mathcal{D}[c_k])\}$ and $c^+ \cdot \rho = c\rho + \rho c^\dagger$.

Graphically: initial state – propagators – operator insertions



⚠ Ignores the singular $\delta(t_i - t_j)$ equal point contributions

Close earlier results:
Zoller & Gardiner 1995, *Lecture Notes for the Les Houches Summer School LXIII on Quantum Fluctuations*
Barchielli & Gregoratti 2009, *Quantum trajectories and measurements in continuous time: the diffusive case*
Verstraete & Cirac 2010, *Phys. Rev. Lett.* **104**, 190405

Including imperfections

In practice, signals are smeared (imperfect amplification):

$$I(t) \to I_\varphi = \int \varphi(u)\, I(u)\, \mathrm{d}u$$

The corresponding correlation function:

$$K_{\ell_1,\cdots,\ell_N}(\varphi_1, \cdots, \varphi_N) := \mathbb{E}\left[I_{\ell_1}(\varphi_1) \cdots I_{\ell_N}(\varphi_N)\right].$$

picks up the equal point δ contributions.

They can be included: **straightforward** albeit slightly cumbersome.

Example

Setup: Qubit without proper dynamics ($\mathcal{L} = 0$), continuously measured with $c_1 = \sqrt{\gamma_x}\sigma_x$ (with associated signal $I_1 := I_x$) and $c_2 = \sqrt{\gamma_-}\sigma_- = \sqrt{\gamma_-}(\sigma_x - i\sigma_y)/2$ (with associated signal $I_2 := I_-$).

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General formula gives *e.g.* :

$$K_{x,-}(t_1, t_2) = \frac{\sqrt{\gamma_-\gamma_x}}{2}\,\mathrm{e}^{-\gamma_-|t_2-t_1|/2} \times \left\{\theta_{t_2 t_1} + \theta_{t_1 t_2}\left[\frac{2\gamma_x}{\gamma_- + 2\gamma_x} - \left(z_0 - \frac{\gamma_-}{\gamma_- + 2\gamma_x}\right)\mathrm{e}^{-(\gamma_- + 2\gamma_x)t_1}\right]\right\}$$

with $z_0 = \mathrm{tr}[\sigma_z\rho(0)]$ and $\theta_{t_1 t_2}$ is the Heaviside function

Example

Include a simple imperfection in the detection chain, low pass filter with bandwidth λ:

- Smears the regular part of correlation functions
- Picks the otherwise "hidden" equal point singularities

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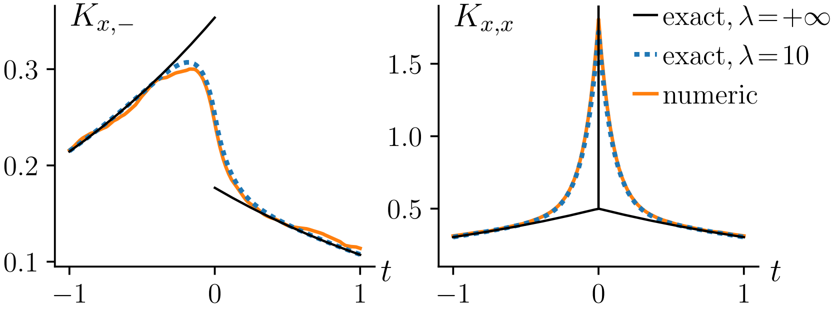

time in units of γ_x

Link with continuous matrix product states

Continuous matrix product states (cMPS)

A variational class of states for $1+1$d quantum field theories. Idem continuous measurement with dictionnary:

- quantum noise $\leftrightarrow$ quantum field
- small system of interest $\leftrightarrow$ variational parameters

Hübener, Mari & Eisert 2013, *Phys. Rev. Lett.* **110** 040401

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Wick's theorem for cMPS

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Continuous measurement translation

All the parameters that can be in principle estimated in the **stationary** state can **generically** be estimated using only 2 and 3 point correlation functions.

Hübener, Mari & Eisert 2013, *Phys. Rev. Lett.* **110** 040401

Summary

There are **smart** ways and **dumb** ways to estimate system parameters:

- Smart is optimal, but difficult, and possibly one day untractable.
- Dumb is suboptimal, but easier to implement

The dumb method relies on the computation of signal N-point functions. The latter can be computed exactly in full generality and including detection imperfections.

Studying and controling quantum systems without reconstructing their state and focusing only on simple functions of the signal may be easier. Being dumb is the future.

Reference: `arXiv:1712.05725`