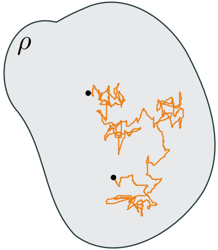
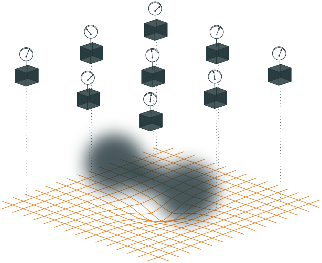
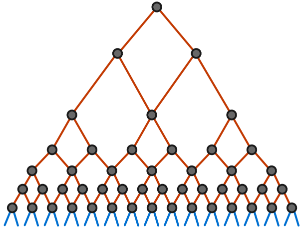
Pulling 3 Threads in Quantum Mechanics

Observation – Unification – Many-body

Antoine Tilloy
Max Planck Institute of Quantum Optics, Garching, Germany







Seminar for the Physics Department
École Polytechnique, Palaiseau, France
September 25th, 2018

Alexander von Humboldt
Stiftung/Foundation

MPQ

3 questions in quantum mechanics that drive me

Observation

How to measure and control quantum systems?

- Fundamentally
- Theoretically FAPP
- In real life

Unification

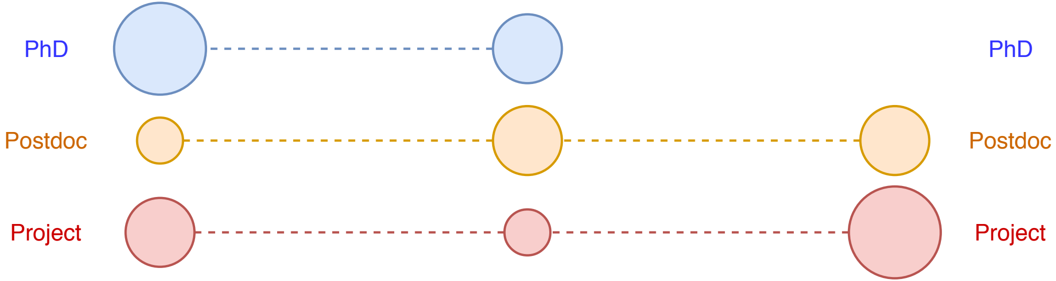
How to unify quantum mechanics and gravity

- Is gravity quantum?
- Clarify with toy models
- Testable predictions

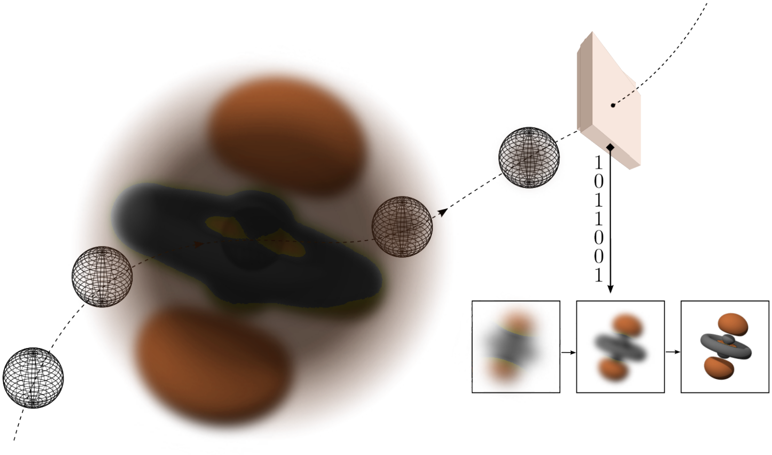
Many-body

How to efficiently manage many-body states

- With tensor networks
- For QFT?



Observation



Motivation

"We know that the moon is demonstrably not there when nobody looks"

David Mermin 1981

Introduction

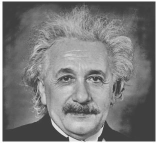
Measurement postulate

For a system "described" by $|\psi\rangle \in \mathscr{H}$ and a measurement of projectors Π_i such that $\sum_i \Pi_i = \mathbb{1}$:

- ♣ **Born rule** : Result i with probability $\mathbb{P}[i] = \langle\psi|\Pi_i|\psi\rangle$
- ♣ **Collapse** : $|\psi\rangle \longrightarrow \frac{\Pi_i|\psi\rangle}{\sqrt{\mathbb{P}[i]}}$

Max Born

John von Neumann



Albert Einstein

John S. Bell

What is a measurement?

- When can the postulate be applied?
- Can measurement be deduced from other postulates?

Introduction

Moving the Heisenberg cut

Limit between the **system**, obeying the Schrödinger equation and the **observer** who can apply the measurement postulate.

Eugene Wigner

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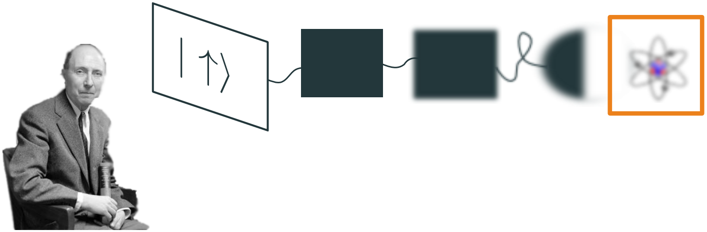

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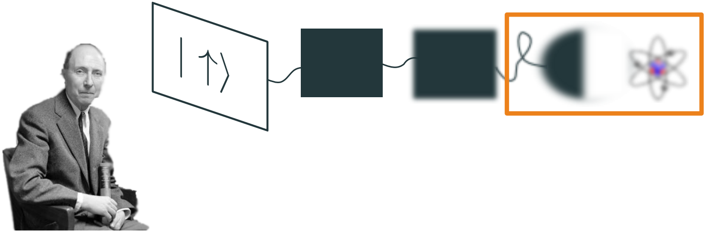

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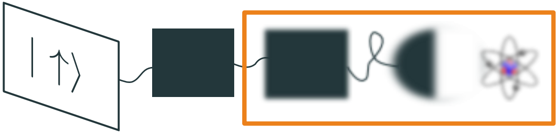

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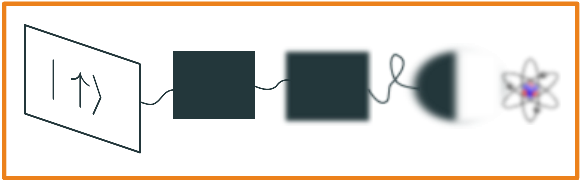

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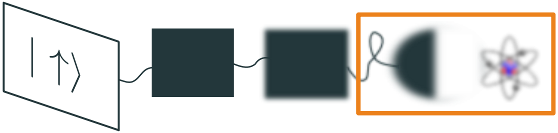
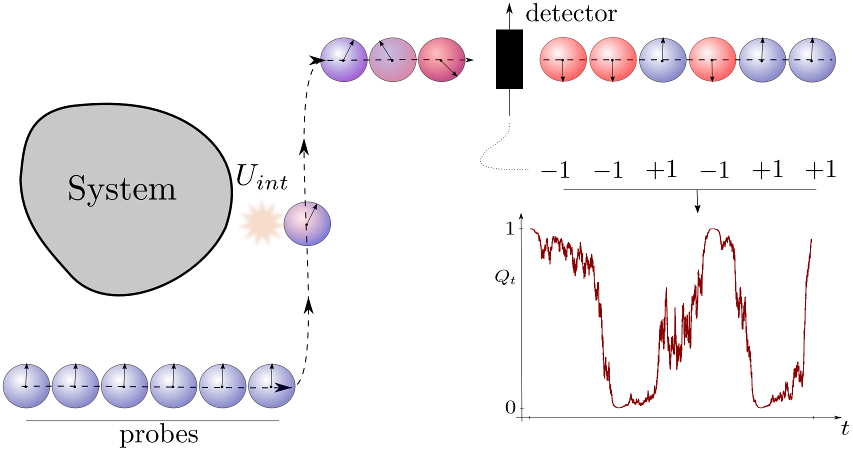


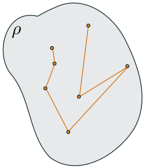
Continuous observation



Repeated interactions

Discrete quantum trajectories

A sequence of $|\psi_n\rangle$ or ρ_n (random) and the corresponding measurement results $\delta_n = \pm 1$.



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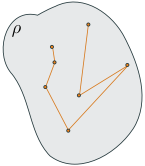

- Make the interaction between system and probe smoother $U_{\text{int}} = \mathbb{1} + i\sqrt{\varepsilon}\, A_{\text{sys}} \otimes B_{\text{probe}}$
- Increase the frequency at which probes are sent: $\tau \propto \varepsilon$

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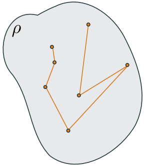

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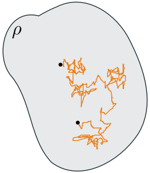
Continuous quantum trajectories

A continuous map $|\psi_t\rangle$ or ρ_t (random) and the corresponding continuous measurement signal $y_t \propto \sqrt{\varepsilon} \sum_k \delta_k$. Typically:

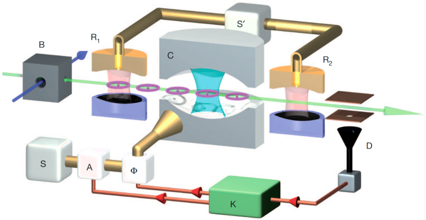
$$\mathrm{d}|\psi_t\rangle = \left[-iH\,\mathrm{d}t + \sqrt{\gamma}(A - \langle A\rangle)\mathrm{d}W_t - \frac{\gamma}{2}(A - \langle A\rangle)^2\,\mathrm{d}t\right]|\psi_t\rangle$$

where W_t Brownian ⚠Essentially a central limit theorem result ⚠

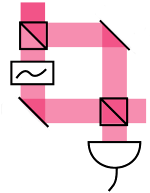


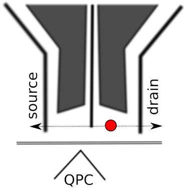
In practice

- Discrete situations "LKB style", with **actual** repeated interactions



- Almost "true" continuous measurement settings (quantum optics, quantum dots)



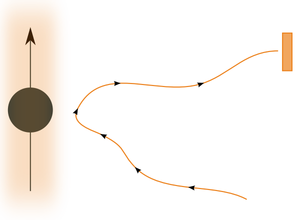


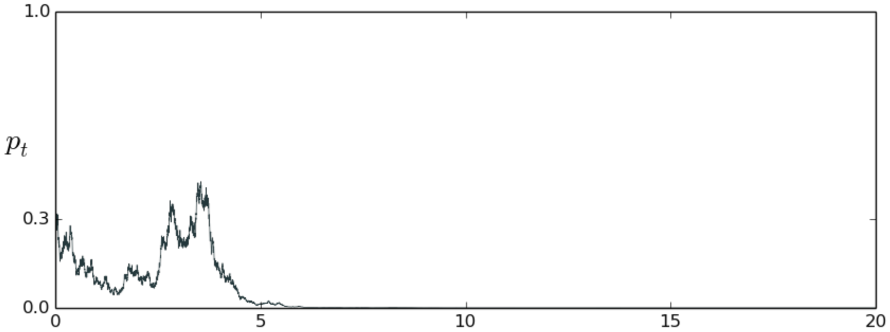
Example 0

Situation considered

Pure continuous measurement of a qubit:

- for the population: $p_t = |\langle\uparrow|\psi_t\rangle|^2$
- one can show: $\mathrm{d}p_t = \sqrt{\gamma}\, p_t(1-p_t)\,\mathrm{d}W_t$





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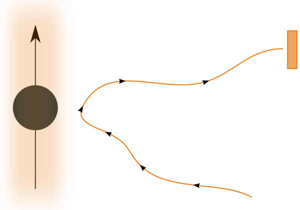

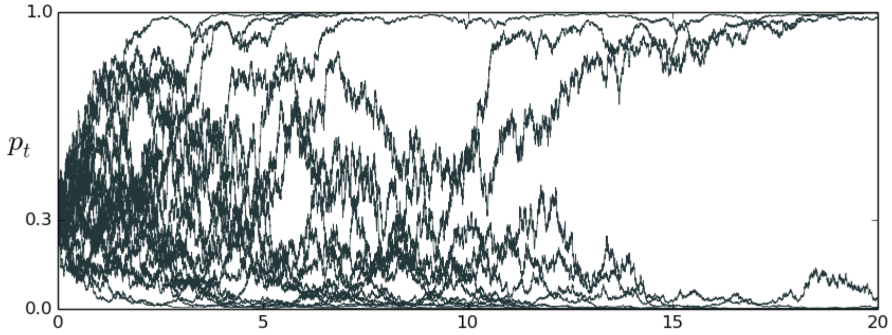

Questions

Measurement is now dynamical with a time scale γ^{-1}. Hence one can:

- Optimize it
- Study its competition with (few-body) unitary dynamics
- Exploit it for real-time "soft" control

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- ♣ Optimize it
- ♣ Study its competition with (few-body) unitary dynamics
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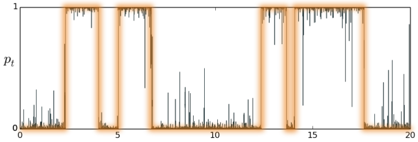
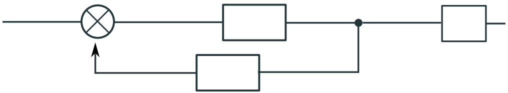
Strong continuous observation $\gamma \gg \omega_i$

- Non-demolition measurement
- Quantum jumps
- Quantum spikes

Weak continuous observation $\gamma \sim \omega_i$

- Optimization
- Control
- Continuous quantum error correction



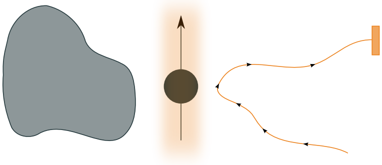


Strong measurement limit: example 1

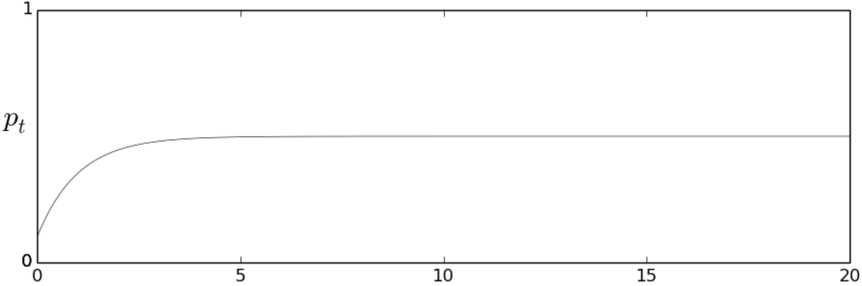
Situation considered

Qubit coupled to a thermal bath

- p_t ground state population
- Thermal bath $p_t \to p^{\text{Boltzmann}}$
- Continuous energy measurement $p_t \to 0$ or 1



No measurement, $\gamma = 0\,\lambda$



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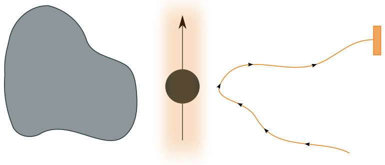
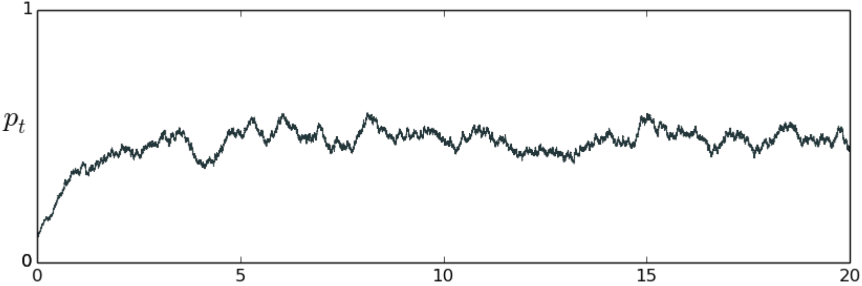


Weak measurement, $\gamma = 0.1\,\lambda$



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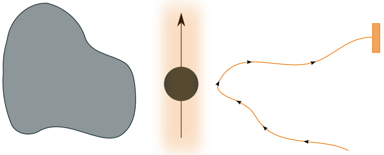
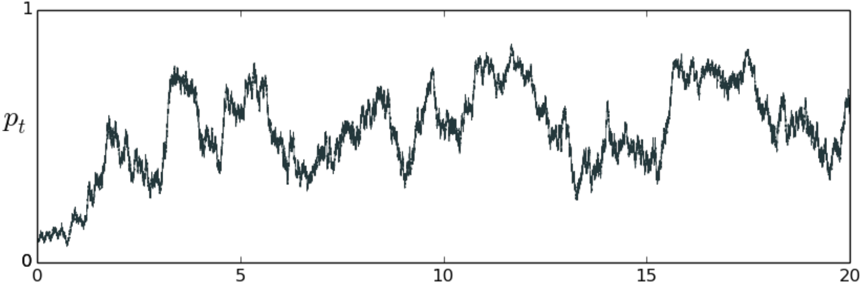


Decent measurement, $\gamma = \lambda$



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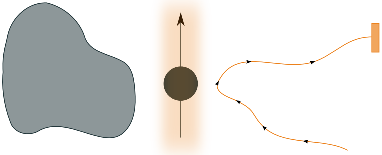
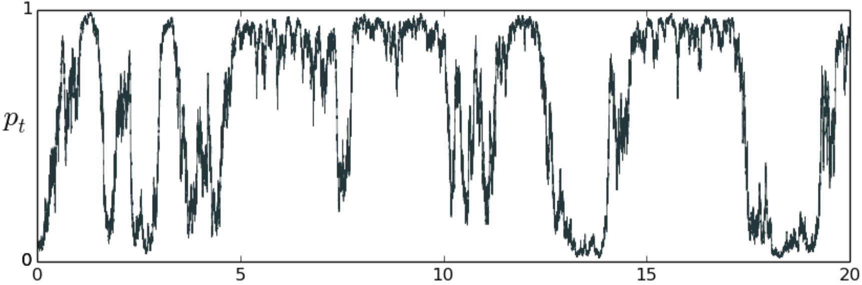


Getting strong measurement, $\gamma = 10\,\lambda$



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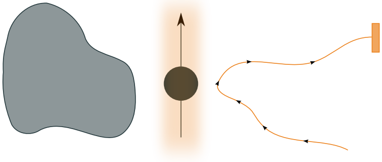
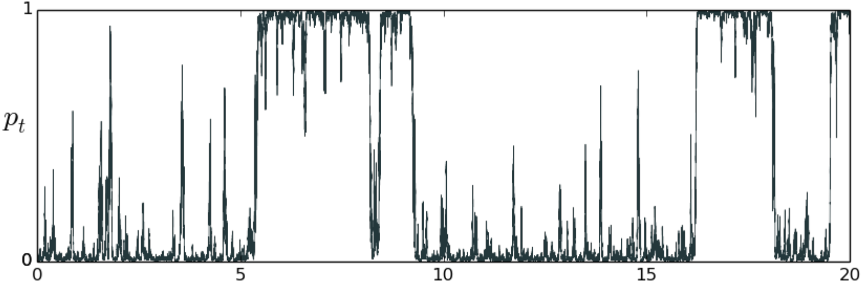


Pretty strong measurement, $\gamma = 100\,\lambda$



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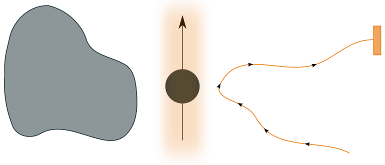
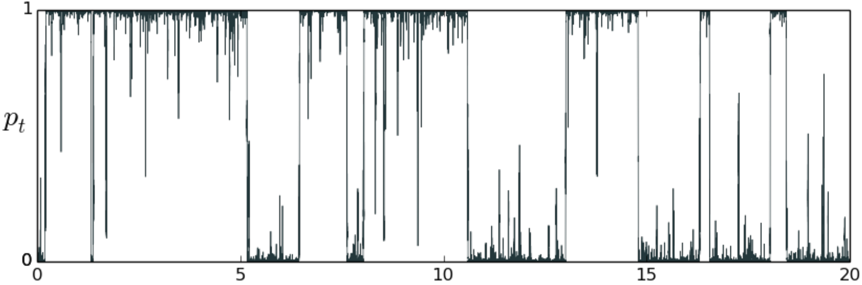


Strong measurement, $\gamma = 1000\,\lambda$



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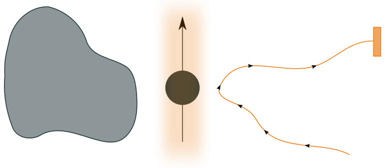
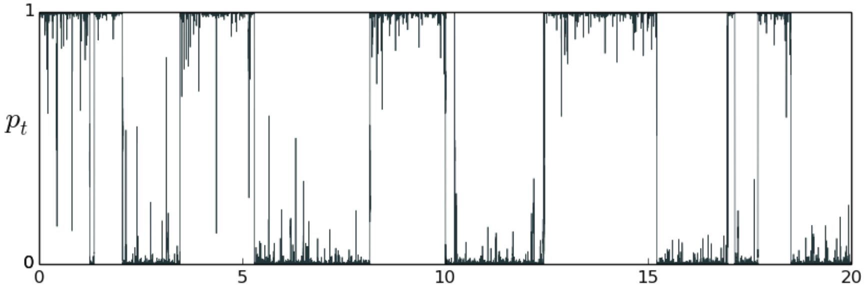


Very strong measurement, $\gamma = 10^4\,\lambda$



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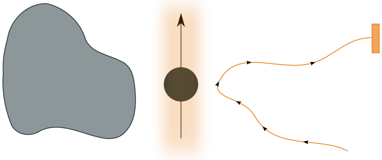
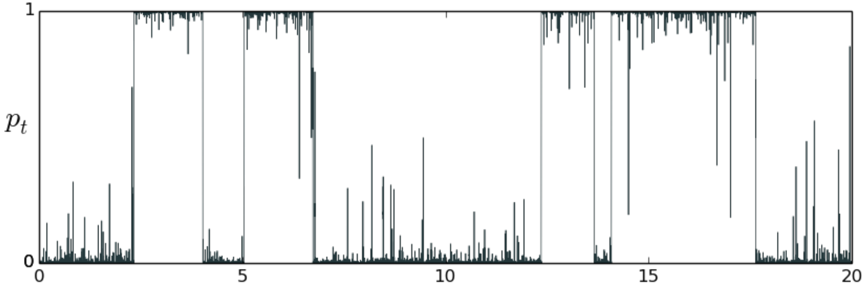


Über strong measurement, $\gamma = 10^5\,\lambda$



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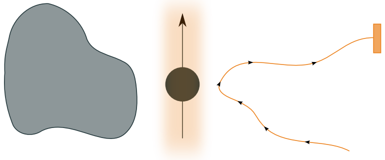

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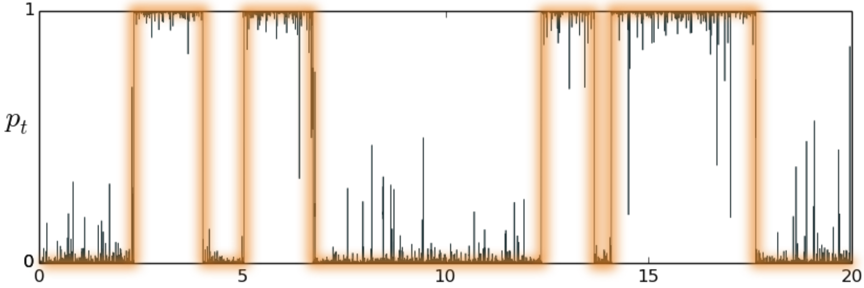
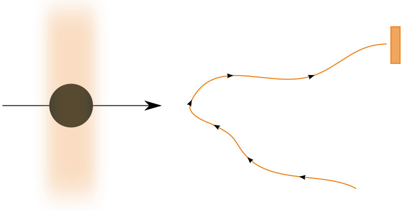


Strong measurement limit: example 2

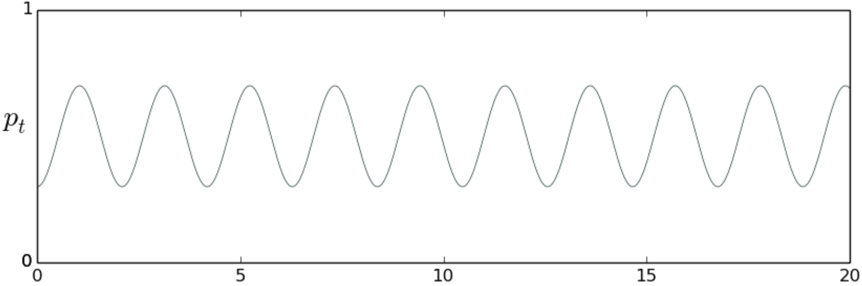
System considered

Qubit in a magnetic field $\perp$ measurement basis

- $p_t = |\langle\psi_t|\uparrow\rangle_z|^2$
- $H = \frac{\omega}{2}\sigma_x$: Rabi oscillations $p_t \sim \cos(\omega t)$
- Measurement $p_t \to 0$ or 1



No measurement, $\gamma = 0\,\omega$



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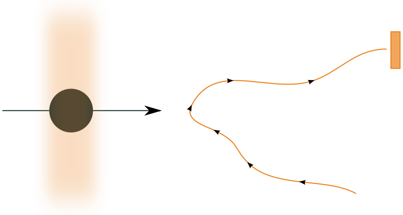
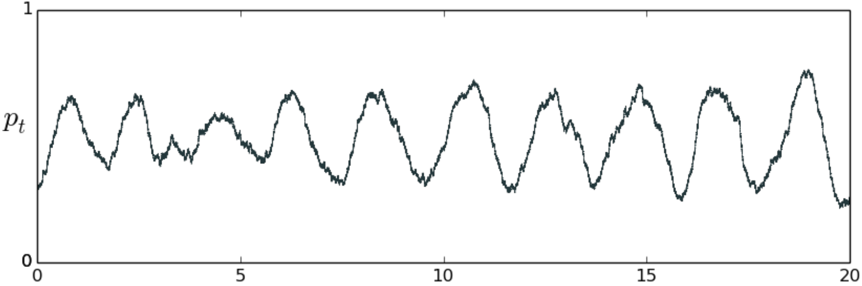


Weak measurement, $\gamma = 0.1\,\omega$



Strong measurement limit: example 2

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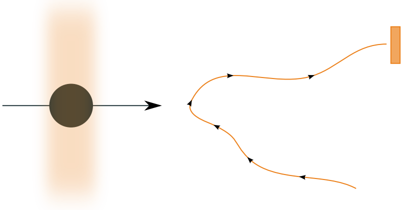
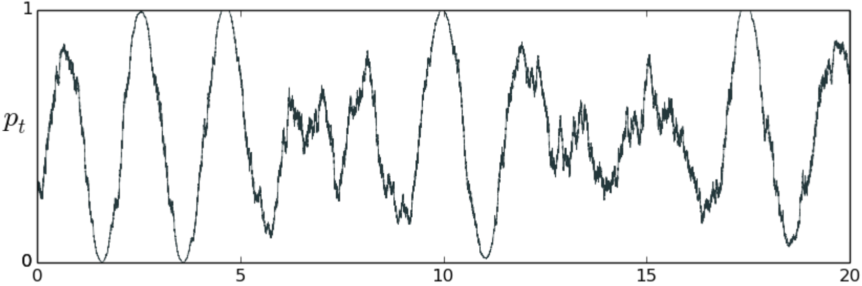


Decent measurement, $\gamma = \omega$



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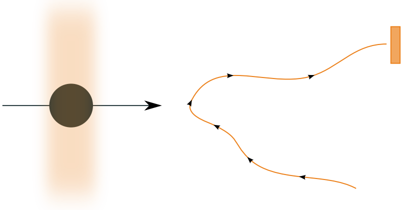
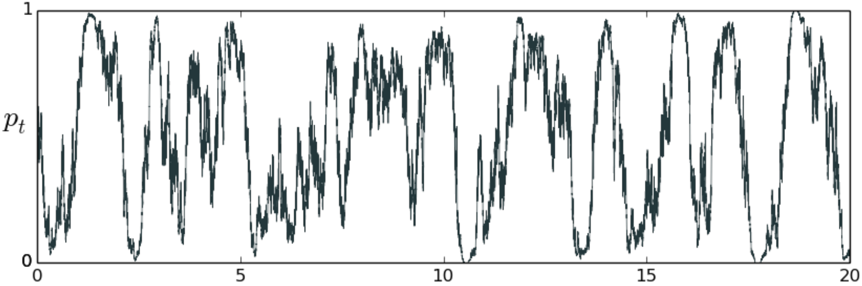


Getting strong measurement, $\gamma = 10\,\omega$



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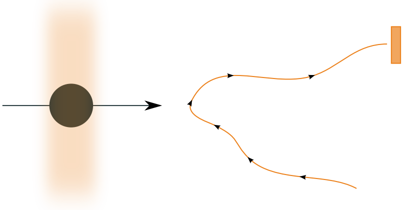
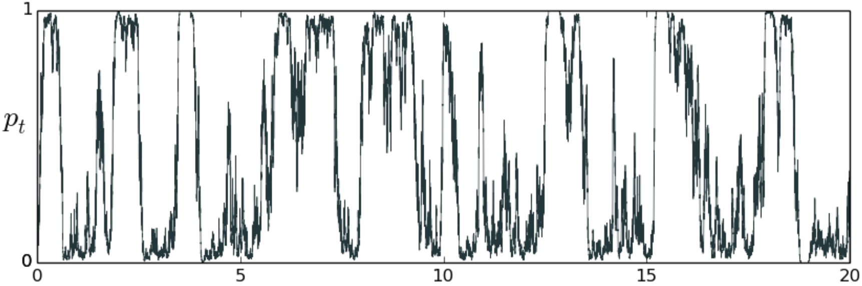


Pretty strong measurement, $\gamma = 30\,\omega$

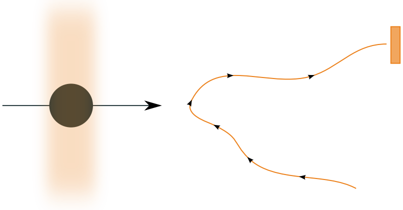


Strong measurement limit: example 2

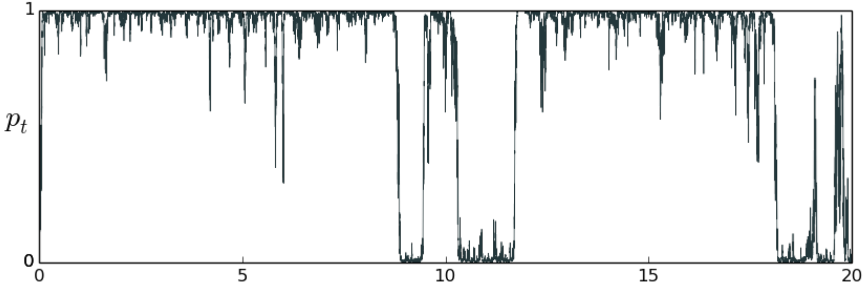
System considered

Qubit in a magnetic field $\perp$ measurement basis

- $p_t = |\langle \psi_t | \uparrow \rangle_z|^2$
- $H = \frac{\omega}{2}\sigma_x$: Rabi oscillations $p_t \sim \cos(\omega t)$
- Measurement $p_t \to 0$ or 1



Strong measurement, $\gamma = 100\,\omega$



Strong measurement limit: example 2

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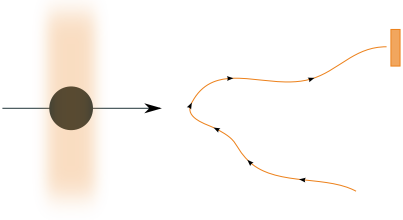
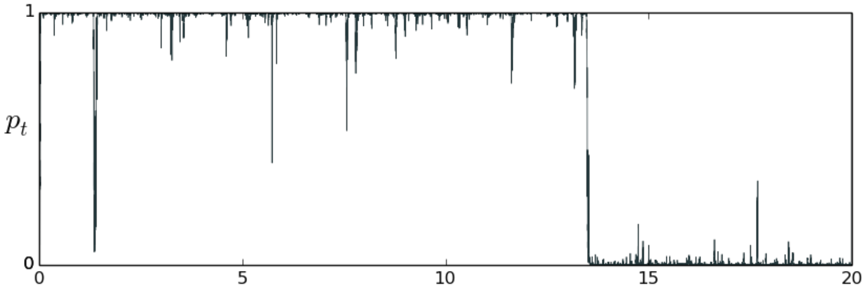


Very strong measurement, $\gamma = 300\,\omega$



Strong measurement limit: example 2

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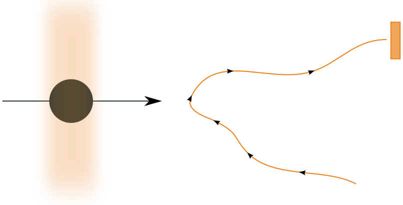
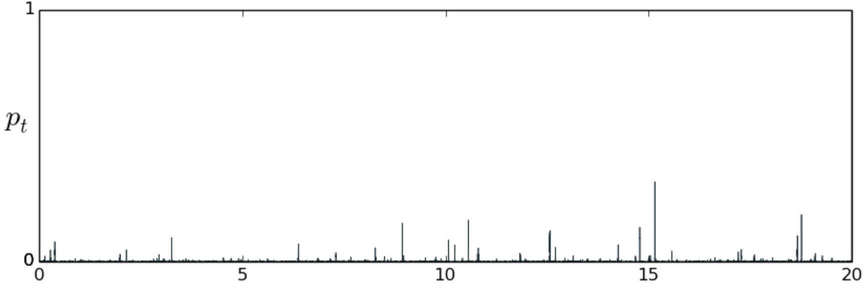


Über strong measurement, $\gamma = 1000\,\omega$



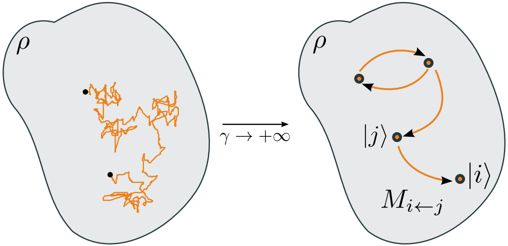
Theorem: jumps

1. Markovian evolution $\mathcal{L}(\rho_t) = L(\rho_t) - i\,[H, \rho_t]$
2. Continuous measurement of $\mathcal{O} = \sum_k \lambda_k |k\rangle\langle k|$

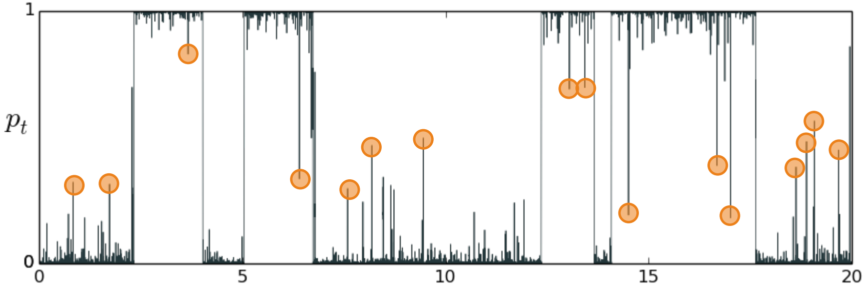
Quantum jumps

When $\gamma \to +\infty$, ρ_t converges to a **Markov chain** with transition matrix M:

$$M_{i\leftarrow j} = \overbrace{L^{ii}_{jj}}^{\text{“incoherent” contribution}} + \underbrace{\frac{1}{4\gamma}\left|\frac{H_{ij}}{\lambda_i - \lambda_j}\right|^2}_{\text{“incoherent” contribution}}$$



A subtlety: spikes



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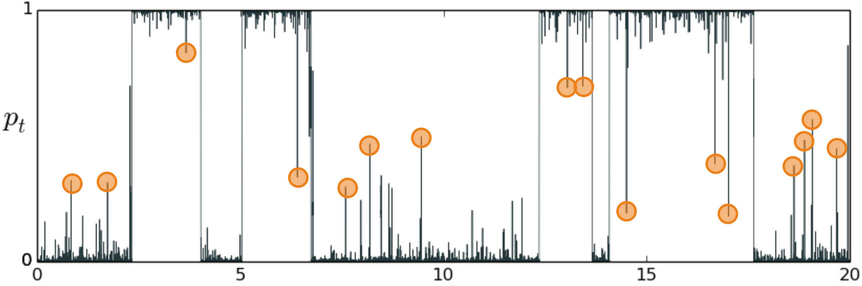


Spikes:

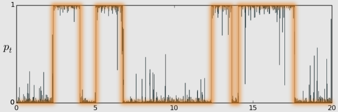
- Remain in the limit
- Are Levy distributed
- Are univeral
- Are experimentally relevant (e.g. for control)

Carrying computations rigorously, one discovers things people did not expect and thought were experimental mistakes

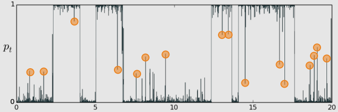
Some results

Strong continuous measurement

1. Jumps



2. Spikes



- ◇ M Bauer, D Bernard, AT JPA 2015
- ◇ AT, M Bauer, D Bernard PRA 2015
- ◇ M Bauer, D Bernard, AT JPA 2016

Others

1. Control
   - ◇ A T, M Bauer, D Bernard EPL 2014
2. Optimal measurement
   - ◇ AT, PRA 2016
3. Exact results
   - ◇ AT, PRA-Rapid 2018
4. Non-Markovian exploration
   - ◇ AT, Quantum 2017
5. Many-body exploration
   - ◇ X Cao, AT, A De Luca, 2018

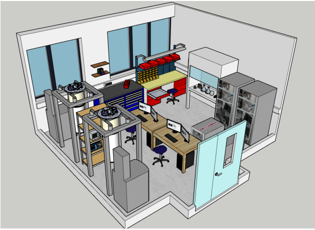
Future

Fast transition in the field in the last $2-3$ years: **new questions**

Applications

Are there obvious questions on the standard theory?

- Theory to experiments
- Experiments to theory



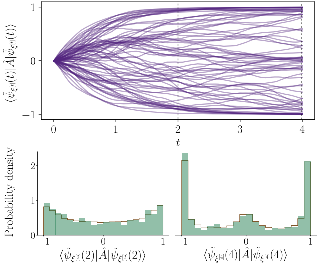
QCMX: Bretheau & Pillet

♠ `Exact signal correlators`
AT, PRA-Rapid 2018

Non-Markovianity

How to include it in the theory?

- N-M feedback
- N-M measurement

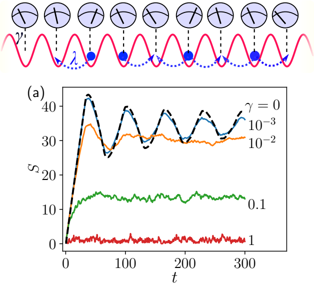


♠ `Non-Markovian Monte-Carlo`
AT, Quantum 2017

Many-body

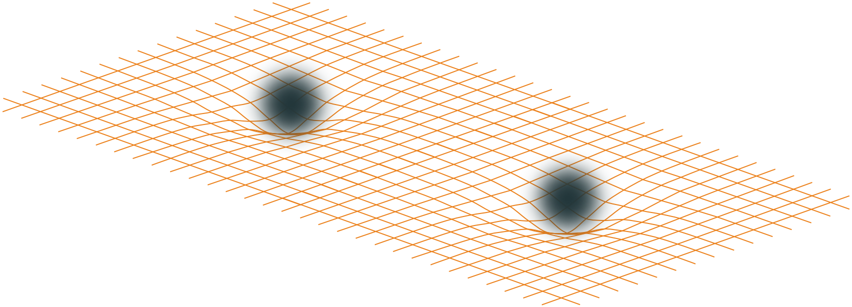
Joining measurement and MB dynamics

- For integrable models
- KPZ universality class?



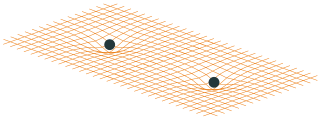
♠ `arXiv:1804.04638`
X Cao, AT, A De Luca

Unification



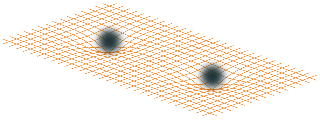
Prolegomena

Classical gravity



- **Matter** is classical
- Spacetime is classical

Semiclassical gravity



- **Matter** is quantum
- Spacetime is classical

Fully quantum gravity

- **Matter** is quantum
- Spacetime is quantum

◇ No experimental evidence for the quantization of gravity

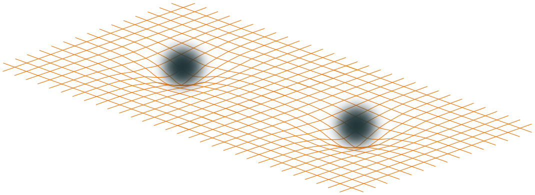
◇ Is semi-classical gravity really impossible?

◇ Can we construct simple toy models clarifying the alleged problems?

"Standard" semi-classical gravity

A semi-classical theory of gravity tells 2 stories:

1. **Quantum matter** moves in a curved classical space-time
2. The classical space-time is curved by **quantum matter**



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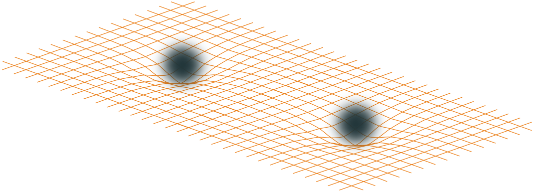


1 is known (QFTCST), **2** is not

The crucial question of semi-classical gravity is to know how quantum matter should source curvature.

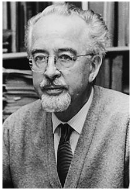
Møller-Rosenfeld semi-classical gravity

Mean-field prescription

The **choice** of Møller and Rosenfeld it to take:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 8\pi G \langle \hat{T}_{\mu\nu} \rangle$$

$\rightarrow$ source gravity via expectation values



Christian Møller

Leon Rosenfeld

There are:

- **technical relativistic** difficulties [renormalization of $\langle T_{\mu\nu} \rangle$]
- **conceptual non-relativistic** difficulties [Born rule, signalling, $\cdots$].

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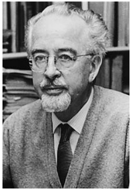

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There are:

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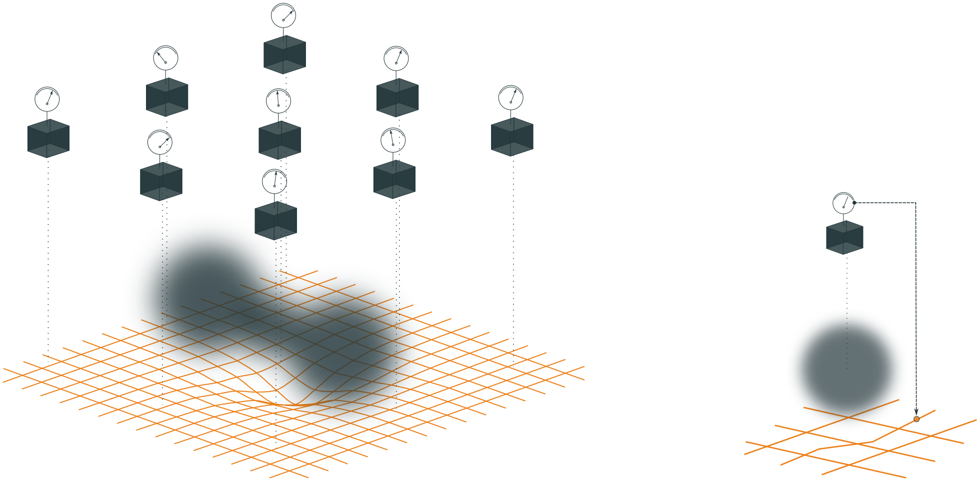
- **conceptual non-relativistic** difficulties [Born rule, signalling, $\cdots$].

Situation

Semiclassical gravity looks impossible even in the Newtonian regime:

♠ What can source the gravitational field if not $\langle \cdot \rangle$? ♠

“Intuition pump” solution



“There are detectors in space-time measuring the mass density continuously and curving space-time accordingly.” → this is why it works

Results (in a nutshell)

Standard quantum feedback like computations give for $\rho_t = \mathbb{E}[|\psi_t\rangle\langle\psi_t|]$:

$$\partial_t \rho_t = -i\left[H_0 + \frac{1}{2}\iint \mathrm{d}x\mathrm{d}y\,\mathscr{V}(x,y)\widehat{M}(x)\widehat{M}(y), \rho_t\right] - \frac{1}{8}\iint \mathrm{d}x\mathrm{d}y\,\mathscr{D}(x,y)\left[\widehat{M}(x),\left[\widehat{M}(y),\rho_t\right]\right],$$

with the **gravitational pair-potential** $\mathscr{V} = \left[\frac{4\pi G}{\nabla^2}\right](x,y) = -\frac{G}{|x-y|}$,
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"Hard" results

- First functioning class of models
  AT, L. Diósi PRD 2016
- Principle to reduce to one model
  AT, L. Diósi PRD 2017

Pedagogical formulation

- Model without Itô calculus
  AT, PRD-Rapid 2018
- General perspective
  AT, `arXiv:1802.03291`

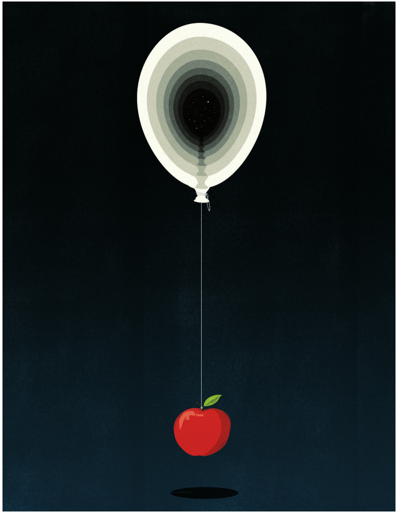
Should one believe in it?

Antoine, do you seriously believe the world is like in your theory?

Sheldon Goldstein

I bet 99 to one that the outcome will be consistent with gravity having quantum properties.

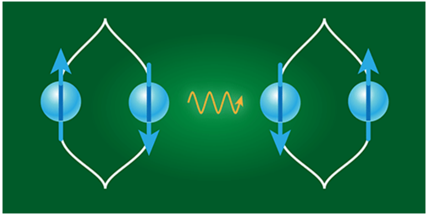
Carlo Rovelli



NewScientist — 14 April 2018

Future

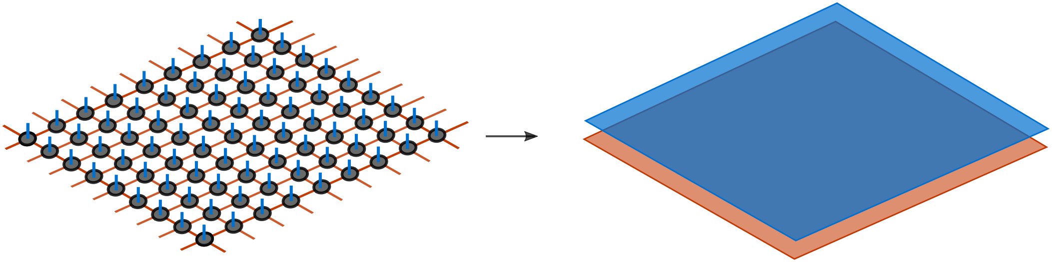
We will know during my lifetime if gravity is a quantum force



Bose *et al.* PRL 2017

- Self-heating computations for neutron stars
- Opportunistic attack of other foundations problems

Many-body: tensor network states



Problem

Many-body states are complicated.

$$|\psi\rangle = \sum_{i_1, i_2, \cdots, i_n} c_{i_1, i_2, \cdots, i_n} |i_1, \cdots, i_n\rangle$$

2^n parameters $c_{i_1, i_2, \cdots, i_n}$.

Typical many-body Hamiltonians are simple.

$$H = \sum_{k=1}^{n} h_k$$

$\sim$ const $\times$ n parameters.

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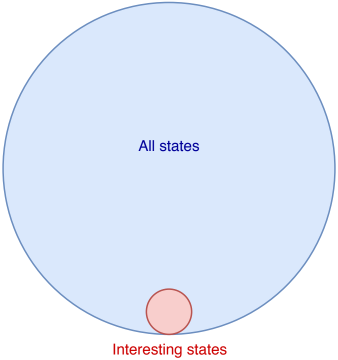

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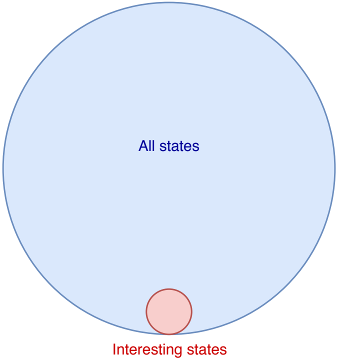


Variational optimization

To find the ground state:

$$|\text{ground}\rangle = \min_{|\psi\rangle \in \mathscr{S}} \frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle}$$

Can we find a subspace $\mathscr{S}$ s. t.:

- $|\mathscr{S}| \propto n^k \ll e^n$
- $\mathscr{S}$ approximates well interesting states
- *bonus* $\langle\psi|\mathcal{O}(x)|\psi\rangle$ is computable

An idea popular in many fields

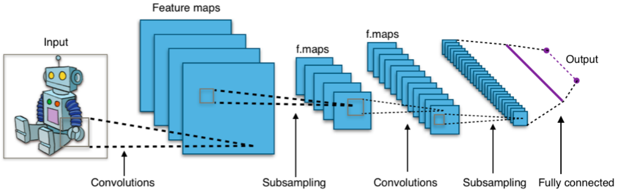
- **Mean field** approximation (of which TNS are an extension)

$$\psi(x_1, x_2, \cdots, x_n) = \psi_1(x_1)\,\psi_2(x_2)\cdots\psi_n(x_n)$$

- Special variational wave functions in **Quantum chemistry** (whole industry of ansatz)
- **Moore-Read wavefunctions** in the study of the quantum Hall effect

$$\psi(x_1, x_2, \cdots, x_n) = \left\langle \hat{\phi}(x_1)\hat{\phi}(x_2)\cdots\hat{\phi}(x_n)\right\rangle_{\mathrm{CFT}}$$

- Fully connected and convolutional **neural networks** used in machine learning



Matrix product states

$$|\psi\rangle = \sum_{i_1, i_2, \cdots, i_n} c_{i_1, i_2, \cdots, i_n} |i_1, \cdots, i_n\rangle$$

Matrix Product States (MPS)

$$|A, L, R\rangle = \sum_{i_1, i_2, \cdots, i_n} \langle \mathrm{L}|A_{i_1}(1) A_{i_2}(2) \cdots A_{i_n}(n)|\mathrm{R}\rangle \, |i_1, \cdots, i_n\rangle$$

- A_i are $D \times D$ complex matrices
- A is a $2 \times D \times D$ tensor $[A_i]_{k,l}$
- $|L\rangle$ and $|R\rangle$ are D-vectors.

Remark: actually equivalent with the density matrix renormalization group (DMRG)

◊ $n \times 2 \times D^2$ parameters instead of 2^n

◊ D is the **bond dimension** and encodes the size of the variational class

Graphical notation

$$|A, L, R\rangle = \sum_{i_1,i_2,\cdots,i_n} \langle L|A_{i_1}(1)A_{i_2}(2)\cdots A_{i_n}(n)|R\rangle \; |i_1,\cdots,i_n\rangle$$

Notation: $[A_i]_{\mathbf{k,l}} =$  and k —— $l = \sum \delta_{k,l}$ gives:

$|A, L, R\rangle =$ 

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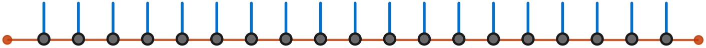
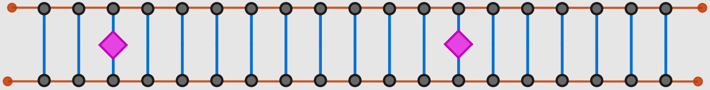
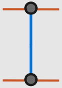
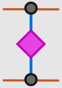
$|A, L, R\rangle =$ 

Example: computation of correlations

$\langle A| \mathcal{O}(i_k) \mathcal{O}(i_\ell) |A\rangle =$ 

can be done by iteration 2 maps:

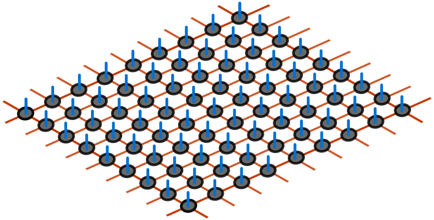
$\Phi =$  and $\Phi_{\mathcal{O}} =$ 

The contraction for a $d = 1$ system, can be seen as an open-system dynamics in $d = 0$.

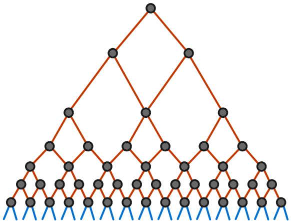
Generalizations: different tensor networks

Matrix Product States (MPS)

Projected Entangled Pair States (PEPS)



Multi-scale Entanglement Renormalization Ansatz (MERA)



Some facts

A list of theorems [very colloquially]:

- **Expressiveness** [trivial] Tensor Network States cover $\mathscr{H}$ when $D \propto 2^n$
- **Area law** The entanglement of a subregion of space scales as its area for a TNS
- **Efficiency** [gapped] Matrix Product States approximate well the ground states of gapped systems in 1 spatial dimension
- **Efficiency** [critical] Multi-scale Entanglement Renormalization Ansatz (MERA) approximate well the ground states of critical systems in 1 spatial dimension.
- **Symmetries** Physical symmetries can be implemented locally on the bond space
- **Inverse problem** TNS are the ground state of a local parent Hamiltonian

Successes and limits

Successes: why the deserved hype

- ♡ Arbitrary precision for $1d$ quantum systems
- ♡ Classification of topological phases in $1d$ and $2d$
- ♡ Progress on non-Abelian lattice Gauge theories
- ♡ AdS/CFT toy models

Limits: why it is overhyped

- ♠ Hard to contract in $d \geqslant 2$
- ♠ No continuum limit in $d \geqslant 2$
- ♠ Lack of analytic techniques

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Limits: why it is overhyped

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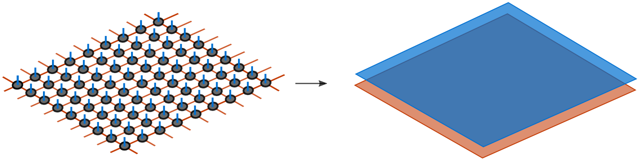
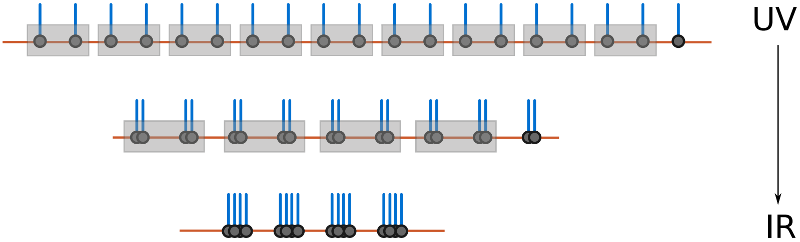
Can one apply tensor network techniques directly in the continuum, to QFT?



Continuous Matrix Product States (cMPS)

Taking the continuum limit of a MPS



Continuous Matrix Product States (cMPS)

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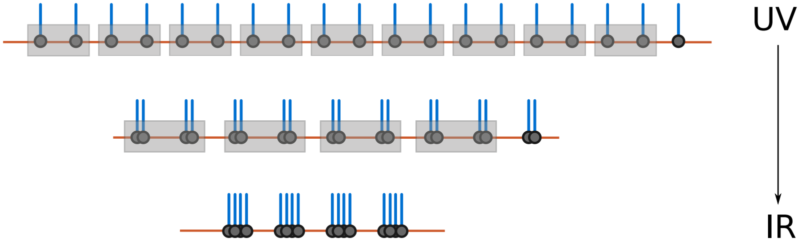
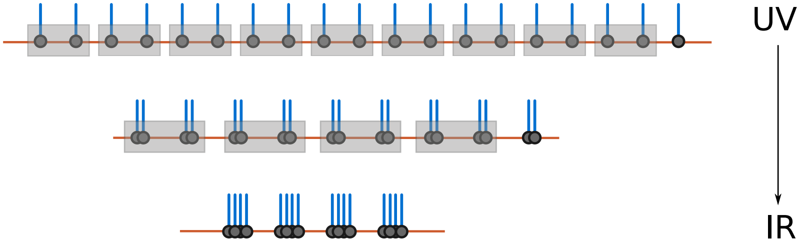


- the bond dimension D stays fixed

Continuous Matrix Product States (cMPS)

Taking the continuum limit of a MPS

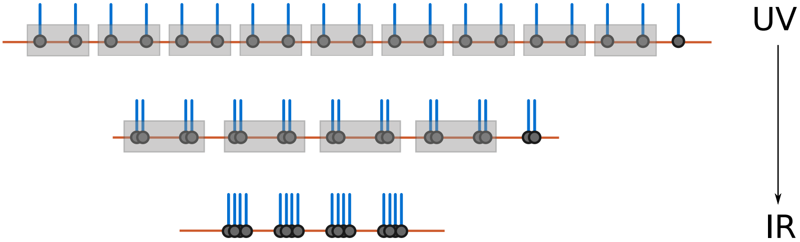


- the bond dimension D stays fixed
- the local physical dimension explodes $\mathbb{C}^2 \otimes \cdots \otimes \mathbb{C}^2 \longrightarrow \mathscr{F}(L^2([x, x+dx]))$.

$\implies$ **Spins** become **fields** – ($\simeq$ central limit theorem $\simeq$ quantum noises $\mathrm{d}\xi$, $\mathrm{d}\xi^\dagger$)

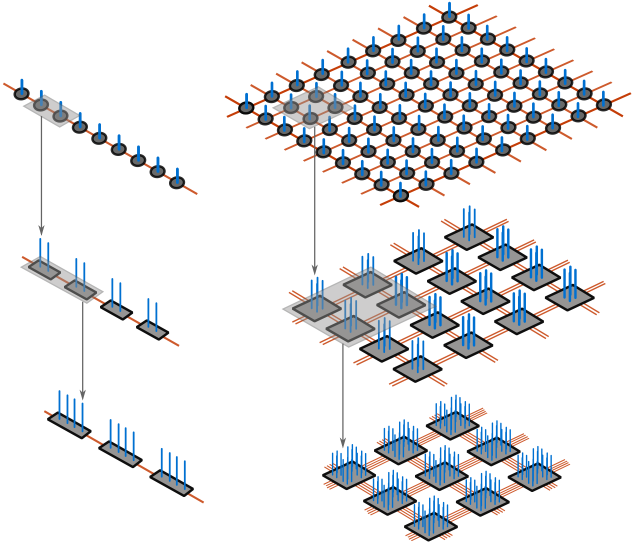
Continuous Matrix Product States (cMPS)

Taking the continuum limit of a MPS

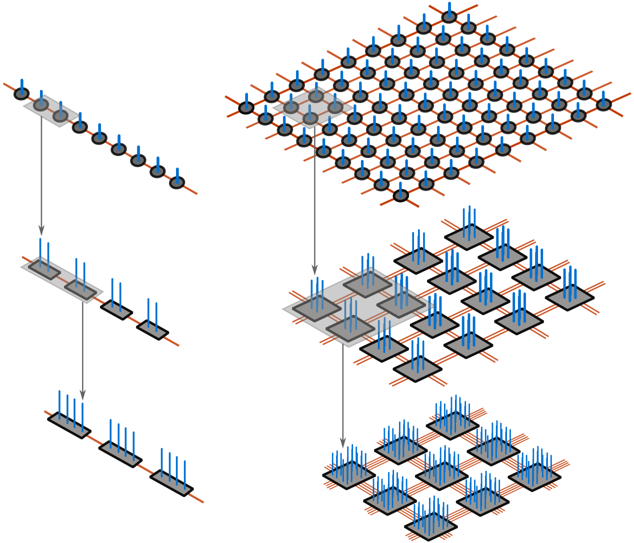


- the bond dimension D stays fixed
- the local physical dimension explodes $\mathbb{C}^2 \otimes \cdots \otimes \mathbb{C}^2 \longrightarrow \mathscr{F}(L^2([x, x+dx]))$.
  $\Longrightarrow$ **Spins** become **fields** – ($\simeq$ central limit theorem $\simeq$ quantum noises $\mathrm{d}\xi$, $\mathrm{d}\xi^\dagger$)
- A cMPS is a quantum field state parameterized by finite dimensional matrices:
  $|Q, R, \omega\rangle = \langle \omega_L | \mathcal{P} \exp \left\{ \int_0^L dx \; Q(x) \otimes \mathbb{1} + R(x) \otimes \psi^\dagger(x) \right\} |\omega_R\rangle \; |0\rangle$

Continuous Tensor Networks: blocking



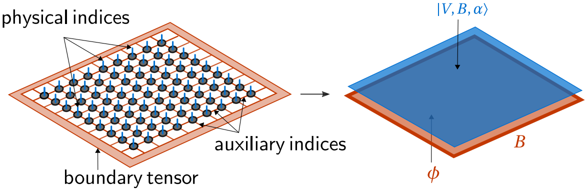
Continuous Tensor Networks: blocking



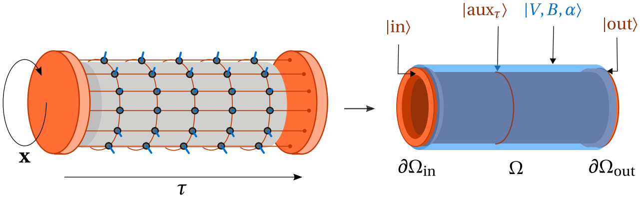
Upon blocking:

- ♣ The **physical** Hilbert space dimension d increases (idem cMPS $\Longrightarrow$ physical field)
- ♣ The **bond** dimension D increases too

Idea: QFT states from classical random fields in the same dimension



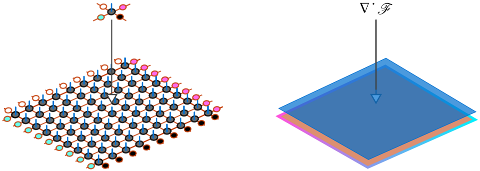
$$|V,B,\alpha\rangle = \int \mathcal{D}\phi\, B(\phi|_{\partial\Omega}) \exp\left\{-\int_\Omega d^d x\, \frac{1}{2}\sum_{k=1}^{D} [\nabla\phi_k(x)]^2 + V[\phi(x)] - \alpha[\phi(x)]\,\psi^\dagger(x)\right\} |0\rangle$$



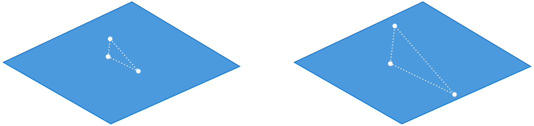
arXiv:1808.00976

Applications and future

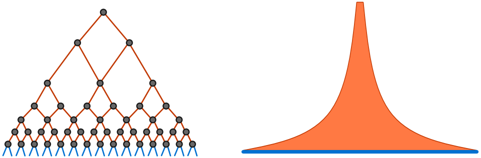
Develop this technology



1. ***Symmetries***



2. ***Renormalization***



3. ***AdS/CFT toys***

Teaching philosophy

- Peculiar students (good level, low drive) have special needs
- What is important for students is not (necessarily) related to my research
- Teaching helps learning
- Higher level popularization and outreach are important

Summary

Continuous measurement theory

Solid mathematics answering semifundamental questions and helping experimentalists on campus.

Alternative semiclassical gravity

An application of the previous theoretical toolbox to deep questions at low added cost

Continuous tensor network states

An ongoing expansion of a powerful mathematical method to the realm of Quantum Field Theory

And hopefully 1 or 2 more columns in the next 5 years...

Technical complements on strong continuous measurements

Theorem: jumps

$$M_{i\leftarrow j} = \overbrace{L^{ii}_{jj}}^{\text{``incoherent'' contribution}} + \underbrace{\frac{1}{4\gamma}\left|\frac{H_{ij}}{\lambda_i - \lambda_j}\right|^2}_{\text{``incoherent'' contribution}}$$

Consequences:

- Gives a signature of the underlying process enabling the transitions: **coherent** vs **incoherent**
- Cannot be reproduced by projective measurements because: $|\lambda_i - \lambda_j| \neq \text{const } \forall i, j$
- Can be used for minimalist control using solely γ (`arXiv:1404.7391`)

Extensions

- Several commuting observables $\mathcal{O}_\ell$
- Repeated imperfect measurements instead of continuous

Jumps: proof

Standard small noise expansion techniques are useless in this context

Idea of the proof
Perturbation theory at the level of the Fokker-Planck equation for ρ_t:

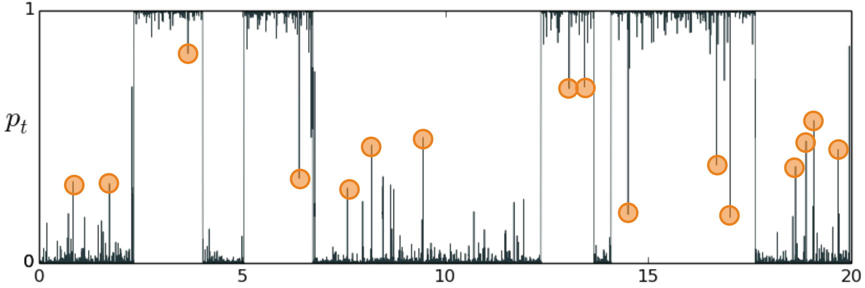
$$\partial_t \mathcal{P}(\rho) = \mathfrak{D}\mathcal{P}(\rho)$$

where $\mathfrak{D}$ is a differential operator

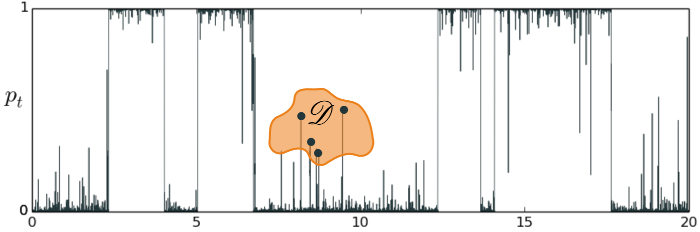
Write $\mathfrak{D} = \gamma\mathfrak{D}_1 + \mathfrak{D}_0$, hence $\mathcal{P}(\rho) = \exp\left(t\gamma\mathfrak{D}_1 + t\mathfrak{D}_0\right)$

- To zeroth order, $\mathcal{P}(\rho) = \exp(t\gamma D_1)$, $\implies$ converges exponentially fast to the kernel of $\mathfrak{D}_1$, i.e. Dirac around **pointer states**
- To next order, $\exp t\mathfrak{D}_0$ gives the transition rates

Spikes



Theorem: spikes



Spike statistics

The number of spikes **starting from** 0 and ending in the domain $\mathscr{D}$ of the plane (t, p) is a Poisson process of intensity $\mu(D)$:

$$\mu = \int_{\mathscr{D}} d\nu \quad \text{with} \quad d\nu = \frac{\lambda}{p^2} dp\, dt$$

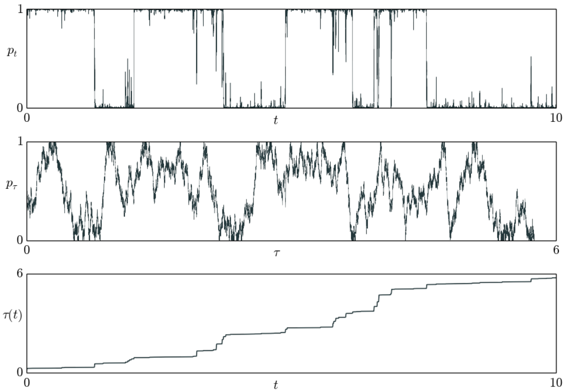
Spikes: idea of the proof

Quickest way: do a ρ dependent time rescaling – arXiv:1512.02861

$$p_t^2(1-p_t)^2\,\mathrm{d}t = \mathrm{d}\tau$$

p_τ has a well defined limit when $\gamma \to +\infty$:

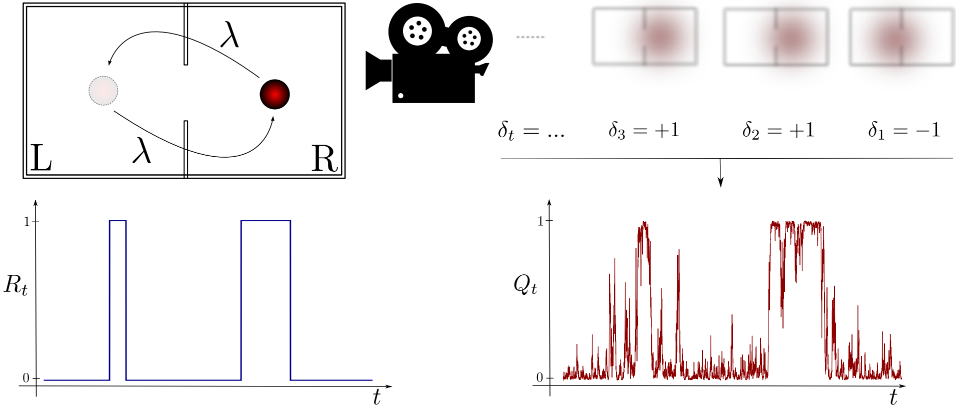
- **Reflected Brownian Motion**



Works only for qubits...

Are spikes real?

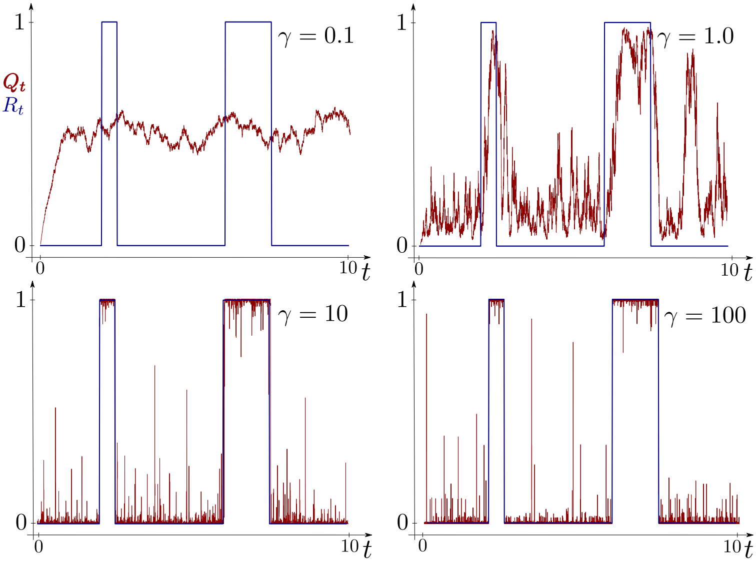
Introduce a classical hidden Markov model:



Yields the same filtering equation as for thermal jumps:

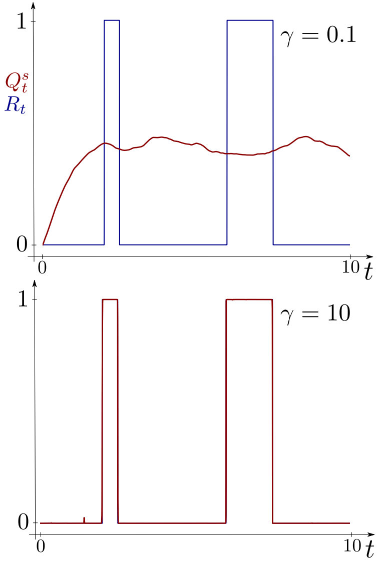
$$dQ_t = \lambda(Q_{eq} - Q_t)\,dt + \sqrt{\gamma}\,Q_t(1 - Q_t)\,dW_t$$

Are spikes real?



Are spikes real?

With (classical) smoothing, i.e. a posteriori estimation:



Technical complements alternative semiclassical gravity

Model

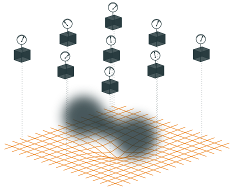
1. **Step 1: continuous mass density measurement**
   We **imagine** that space-time is filled with detectors weakly measuring the mass density:

The equation for matter is now as before with

$$\mathcal{O} \to \hat{M}(x), \ \ \forall x \in \mathbb{R}^3$$

$\gamma \to \gamma(x, y)$ coding detector strength and correlation

and there is a "mass density signal" $\mathsf{S}(x)$ in every point.

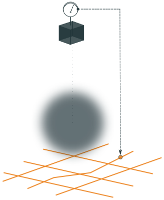


2. **Step 2: Feedback**

We take the mass density signal $\mathsf{S}(x)$ to source the gravitational field φ:

$$\nabla^2 \varphi(x) = 4\pi\, G\, \mathsf{S}(x)$$

which is **formally** equivalent to quantum feedback.



Result

Standard quantum feedback like computations give for $\rho_t = \mathbb{E}[|\psi_t\rangle\langle\psi_t|]$:

$$\partial_t \rho = -i\left[H_0 + \frac{1}{2}\iint \mathrm{d}x\mathrm{d}y\,\mathscr{V}(x,y)\hat{M}(x)\hat{M}(y), \rho_t\right] - \frac{1}{8}\iint \mathrm{d}x\mathrm{d}y\,\mathscr{D}(x,y)\left[\hat{M}(x),\left[\hat{M}(y),\rho_t\right]\right],$$

with the **gravitational pair-potential**

$$\mathscr{V} = \left[\frac{4\pi G}{\nabla^2}\right](x,y) = -\frac{G}{|x-y|},$$

and the **positional decoherence**

$$\mathscr{D}(x,y) = \left[\frac{\gamma}{4} + \mathscr{V}\circ\gamma^{-1}\circ\mathscr{V}^{\top}\right](x,y)$$

Hence the expected pair potential has been generated consistently at the price of more decoherence.

Principle of least decoherence

$$\mathscr{D}(x,y) = \left[\frac{\gamma}{4} + \mathscr{V} \circ \gamma^{-1} \circ \mathscr{V}^{\top}\right](x,y)$$

There is still a (functional) degree of freedom $\gamma(x,y)$:

- Large $\|\gamma\| \implies$ strong "measurement" induced decoherence
- Small $\|\gamma\| \implies$ strong "feedback" decoherence

There is an optimal kernel that minimizes decoherence.

Diagonalizing in Fourier, one gets a global minimum for

$$\gamma = 2\sqrt{\mathscr{V} \circ \mathscr{V}^{\top}} = -2\mathscr{V}$$

Hence:

$$\mathscr{D}(x,y) = -\mathscr{V}(x,y) = \frac{G}{|x-y|}$$

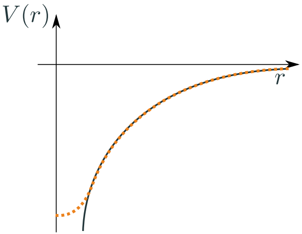
This is just the decoherence kernel of the Diósi-Penrose model (erstwhile heuristically derived)!

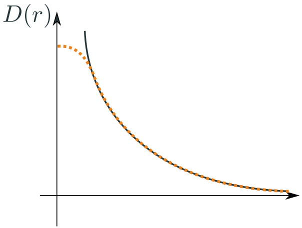
Regularization

Even for the minimal decoherence prescription, the decoherence is **infinite**.
Adding a regulator at a length scale σ has 2 effects:

- It tames decoherence, making it finite
- It regularizes the pair potential $\propto \frac{1}{r}$ for $r \lesssim \sigma$

$\Longrightarrow$ there is a **trade-off**.





Experimentally:

$$\underbrace{10^{-15} m}_{\text{decoherence constraint}} \ll \sigma \leqslant \underbrace{10^{-4} m}_{\text{gravitational constraint}}$$

Importantly $\sigma > \ell_{\text{Compton}} \gg \ell_{\text{Planck}}$.

Experimental final word (for this approach)

PRL 119, 240401 (2017) PHYSICAL REVIEW LETTERS week ending 15 DECEMBER 2017

Spin Entanglement Witness for Quantum Gravity

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Understanding gravity in the framework of quantum mechanics is one of the great challenges in modern physics. However, the lack of empirical evidence has lead to a debate on whether gravity is a quantum entity. Despite varied proposed probes for quantum gravity, it is fair to say that there are no feasible ideas yet to test its quantum coherent behavior directly in a laboratory experiment. Here, we introduce an idea for such a test based on the principle that two objects cannot be entangled without a quantum mediator. We show that despite the weakness of gravity, the phase evolution induced by the gravitational interaction of two micron size test masses in adjacent matter-wave interferometers can detectably entangle them even when they are placed far apart enough to keep Casimir-Polder forces at bay. We provide a prescription for witnessing this entanglement, which certifies gravity as a quantum coherent mediator, through simple spin correlation measurements.

DOI: 10.1103/PhysRevLett.119.240401

Gravitationally-induced entanglement between two massive particles is sufficient evidence of quantum effects in gravity

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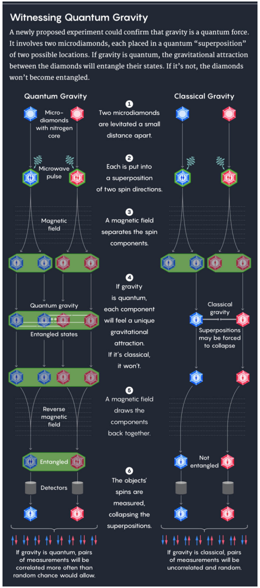
[b]Centre for Quantum Technologies, National University of Singapore

(Dated: December 9, 2017)

All existing quantum gravity proposals are extremely hard to test in practice. Quantum effects in the gravitational field are exceptionally small, unlike those in the electromagnetic field. The fundamental reason is that the gravitational coupling constant is about 43 orders of magnitude smaller than the fine structure constant, which governs light-matter interactions. For example, detecting gravitons – the hypothetical quanta of the gravitational field predicted by certain quantum-gravity proposals – is deemed to be practically impossible. Here we adopt a radically different, quantum-information-theoretic approach to testing quantum gravity. We propose to witness quantum-like features in the gravitational field, by probing it with two masses each in a superposition of two locations. First, we prove that any system (e.g. a field) mediating entanglement between two quantum systems must be quantum. This argument is general and does not rely on any specific dynamics. Then, we propose an experiment to detect the entanglement generated between two masses via gravitational interaction. By our argument, the degree of entanglement between the masses is a witness of the field quantisation. This experiment does not require any quantum control over gravity. It is also closer to realisation than detecting gravitons or detecting quantum gravitational vacuum fluctuations.



Lucy Reading-Ikkanda/Quanta Magazine

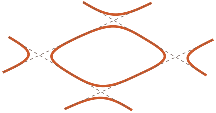
Technical complements on the cTNS construction

Choice of trivial tensor

For **MPS**, not much choice:

For **TNS** in $d \geqslant 2$, many options:

1. Take a δ between all legs $\sim$ GHZ state $T^{(0)} =$ 
   $\implies$ trivial geometry
2. Take two identities $T^{(0)} =$ 
   $\implies$ breakdown of Euclidean invariance
3. Take the sum of pairs of identities in both directions $T^{(0)} =$ 



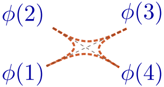
We will consider a softer modification of the first version:

$T^{(0)} \sim$ 

Ansatz

1 – Take a “Trivial” tensor:

$$T^{(0)}_{\phi(1),\phi(2),\phi(3),\phi(4)} =$$



$$\sim \exp\left\{ \frac{-1}{2} \sum_{k=1}^{D} [\phi_k(1) - \phi_k(2)]^2 + [\phi_k(2) - \phi_k(3)]^2 + [\phi_k(3) - \phi_k(4)]^2 + [\phi_k(4) - \phi_k(1)]^2 \right\}$$

The indices ϕ are in $\mathbb{R}^D$ (and **not** $1, \cdots, D$)

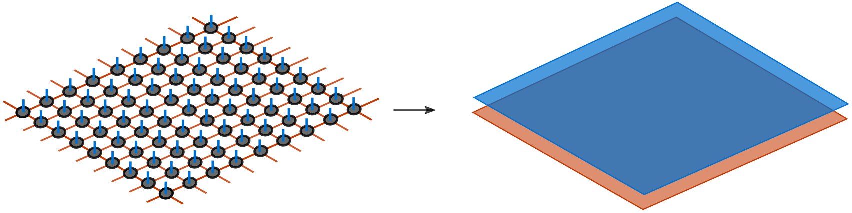
2 – And add a “correction”:

$$\exp\left\{-\varepsilon^2 V\left[\phi(1), \cdots, \phi(4)\right] + \varepsilon^2 \alpha\left[\phi(1), \cdots, \phi(4)\right] \psi^\dagger(x)\right\}$$

3 – Realize tensor contraction = functional integral and trivial tensor gives free field measure.

Functional integral definition

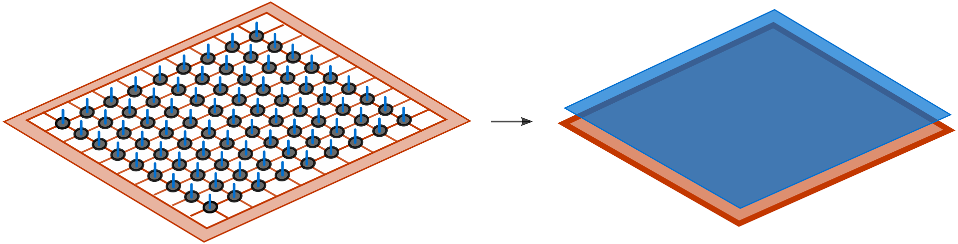
In the continuum limit:



$$|V,\alpha\rangle = \int \mathcal{D}\phi \, \exp\left\{-\int_\Omega \mathrm{d}^d x \, \frac{1}{2}\sum_{k=1}^{D} [\nabla\phi_k(x)]^2 + V[\phi(x)] - \alpha[\phi(x)]\,\psi^\dagger(x)\right\} |0\rangle$$

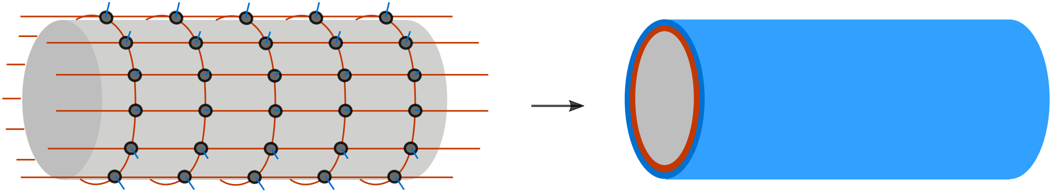
Functional integral definition

In the continuum limit:



$$|V, B, \alpha\rangle = \int \mathcal{D}\phi \; B(\phi|_{\partial\Omega}) \exp\left\{-\int_\Omega \mathrm{d}^d x \; \frac{1}{2}\sum_{k=1}^{D} [\nabla\phi_k(x)]^2 + V[\phi(x)] - \alpha[\phi(x)]\,\psi^\dagger(x)\right\} |0\rangle$$

Operator definition



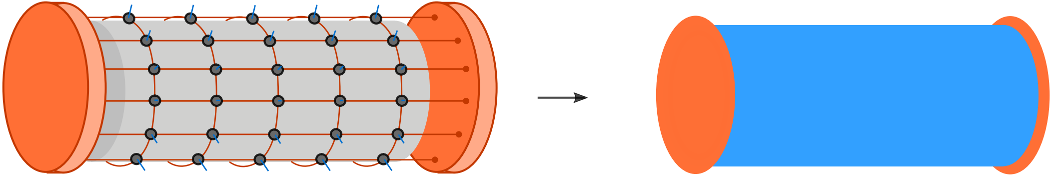
$$|V, \alpha\rangle =$$

$$\mathrm{tr}\left[\mathcal{T}\exp\left(-\int_0^T d\tau \int_S dx\, \frac{\hat{\pi}_k(x)\hat{\pi}_k(x)}{2} + \frac{\nabla\hat{\phi}_k(x)\nabla\hat{\phi}_k(x)}{2} + V[\hat{\phi}(x)] - \alpha[\hat{\phi}(x)]\, \psi^\dagger(\tau, x)\right)\right]|0\rangle$$

where:

- $\hat{\phi}_k(x)$ and $\hat{\pi}_k(x)$ are k independent canonically conjugated pairs of (auxiliary) field *operators*: $[\hat{\phi}_k(x), \hat{\phi}_l(y)] = 0$, $[\hat{\pi}(x)_k, \hat{\pi}_l(y)] = 0$, and $[\hat{\phi}_k(x), \hat{\pi}_l(y)] = i\delta_{k,l}\,\delta(x-y)$ acting on a space of $d-1$ dimensions.

Operator definition



$$|V, B, \alpha\rangle =$$

$$\mathrm{tr}\left[\hat{B}\,\mathcal{T}\exp\left(-\int_0^T d\tau \int_S dx\, \frac{\hat{\pi}_k(x)\hat{\pi}_k(x)}{2} + \frac{\nabla\hat{\phi}_k(x)\nabla\hat{\phi}_k(x)}{2} + V[\hat{\phi}(x)] - \alpha[\hat{\phi}(x)]\,\psi^\dagger(\tau, x)\right)\right]|0\rangle$$

where:

- $\hat{\phi}_k(x)$ and $\hat{\pi}_k(x)$ are k independent canonically conjugated pairs of (auxiliary) field *operators*: $[\hat{\phi}_k(x), \hat{\phi}_l(y)] = 0$, $[\hat{\pi}(x)_k, \hat{\pi}_l(y)] = 0$, and $[\hat{\phi}_k(x), \hat{\pi}_l(y)] = i\delta_{k,l}\,\delta(x-y)$ acting on a space of $d-1$ dimensions.

Expressivity and stability

How big are cTNS?

Stability

The sum of two cTNS of bond field dimension D_1 and D_2 is a cTNS with bond field dimension $D \leqslant D_1 + D_2 + 1$:

$$|V_1, \alpha_1\rangle + |V_2, \alpha_2\rangle = |W, \beta\rangle$$

Expressivity

All states in the Fock space can be approximated by cTNS:

- A field coherent state is a cTNS with $D = 0$
- Stability allows to get all sums of field coherent states

Note: expressivity can also be obtained with $D = 1$ but it is less natural. Flexibility in D makes the expressivity higher for restricted classes of V and α.

Computations

Define generating functional for normal ordered correlation functions

$$\mathcal{Z}_{j',j} = \frac{1}{\langle V,\alpha|V,\alpha\rangle} \langle V,\alpha| \exp\left(\int \mathrm{d}x\, j'(x)\psi^\dagger(x)\right) \exp\left(\int \mathrm{d}x\, j(x)\psi(x)\right) |V,\alpha\rangle$$

Functional integral representation

- Use formula for overlap of field coherent states

$$\langle\beta|\alpha\rangle = \exp\left(\int \mathrm{d}x\, \beta^*(x)\,\alpha(x)\right)$$

- Compute with Gaussian integration + Feynman diagrams or Monte Carlo

Operator representation

Similar to cMPS

- Transfer matrix

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = \mathrm{tr}\left(\Phi_{\mathcal{O}} \cdot e^{-(y-x)T}\Phi_{\mathcal{O}} \cdot \rho_{\mathrm{stat}}\right)$$

with $T = Q \otimes \mathbb{1} + \mathbb{1} \otimes \bar{Q} + R \otimes \bar{R}$ with

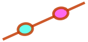
$$Q = -\int \frac{\hat{\pi}_k(x)^2 + [\nabla\hat{\phi}_k(x)]^2}{2} + V(\hat{\phi}(x))$$

and $R \otimes \bar{R} = \int V(\hat{\phi}(x)) \otimes V(\hat{\phi}(x))^\dagger$

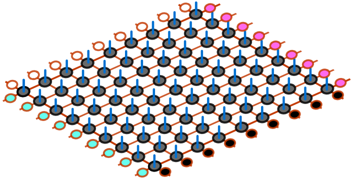
Redundancies

Discrete redundancy

Different elementary tensors are **equivalent**, they give the same state:

when  $=$  and  $=$ 

up to **boundary** terms:

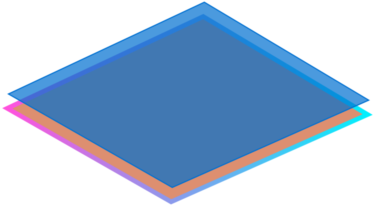


Continuum redundancy

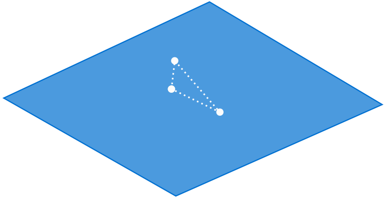
$$V(\phi) \to V(\phi) + \nabla \cdot \mathscr{F}[x, \phi(x)]$$

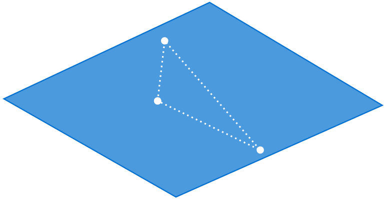
Just Stokes' theorem. If Ω has a boundary $\partial\Omega$:

$$\mathcal{D}[\phi] \to \mathcal{D}[\phi] \exp\left\{\oint_{\partial\Omega} \mathrm{d}^{d-1}x\, \mathscr{F}[x, \phi(x)] \cdot \mathbf{n}(x)\right\}$$



Renormalization





$$C(x_1, \cdots, x_n) = \langle T(1)|\mathcal{O}(x_1)\cdots\mathcal{O}(x_n)|T(1)\rangle,$$

the objective is to find a tensor $T(\lambda)$ of new parameters such that:

$$C(\lambda x_1, \cdots, \lambda x_n) \propto \langle T(\lambda)|\mathcal{O}(x_1)\cdots\mathcal{O}(x_n)|T(\lambda)\rangle.$$

Doable exactly:

$$V \to \lambda^d V \circ \lambda^{\frac{2-d}{2}} \quad \text{and} \quad \alpha \to \lambda^{\frac{d}{2}} \alpha \circ \lambda^{\frac{2-d}{2}}$$

- $d = 2$, All powers of the field in V and α yield relevant couplings
- $d = 3$, The powers $p = 1, 2, 3, 4, 5$ of the field in V yield relevant $\Delta > 0$ couplings. The power $p = 6$ is marginal in V. For α, the powers $p = 1, 2$ are relevant and $p = 3$ is marginal. All other powers are irrelevant.

Getting back cMPS

One can get back cMPS with finite bond dimension by:

1. **Compactification** Take $d-1$ dimensions out of d to be very small

$$|V,B,\alpha\rangle \simeq \mathrm{tr}\left[\hat{B}\,\mathcal{T}\exp\left(-\int_0^T \mathrm{d}\tau \sum_{k=1}^{D} \frac{\hat{P}_k^2}{2} + V[\hat{X}] - \alpha[\hat{X}]\,\psi^\dagger(\tau)\right)\right]|0\rangle$$

$\Longrightarrow$ Hilbert space of a quantum particle in D space dimensions.

2. **Quantization** Take V with D deep minima to force the auxiliary field to take only D possibilities

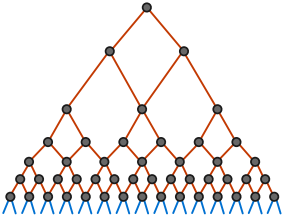
Generalization

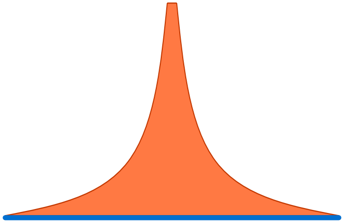
For a general Riemanian manifold $\mathcal{M}$ with boundary $\partial\mathcal{M}$, define:

$$|V, B, \alpha\rangle = \int \mathcal{D}\phi\, B(\phi|_{\partial\mathcal{M}}) \exp\left\{-\int_{\mathcal{M}} \mathrm{d}^d x \sqrt{g}\left(\frac{g^{\mu\nu}\partial_\mu\phi_k\partial_\nu\phi_k}{2} + V[\phi, \nabla\phi] - \alpha[\phi, \nabla\phi]\,\psi^\dagger\right)\right\}|0\rangle$$

i.e. add curvature and possible anisotropies in V and α

Example: $\alpha[x, \phi, \nabla\phi]$ localized on the boundary and hyberbolic metrix g:





$\rightarrow$ **cMERA** in $d-1$ dimensions

Summary

$$|V, B, \alpha\rangle = \int \mathcal{D}\phi \, B(\phi|_{\partial\Omega}) \exp\left\{-\int_\Omega \mathrm{d}^d x \, \frac{1}{2}\sum_{k=1}^{D} [\nabla\phi_k(x)]^2 + V[\phi(x)] - \alpha[\phi(x)]\,\psi^\dagger(x)\right\} |0\rangle$$

Continuous tensor network states are natural continuum limits of tensor network states and natural higher d extensions of continuous matrix product states.

1. Obtained from discrete tensor networks
2. Can be made Euclidean invariant
3. Have functional and operator representations
4. Have a geometrical equivalent of the discrete gauge redundancies
5. Have an exact and explicit renormalization flow