Towards tensor network methods for QFT

and other quantum inquiries

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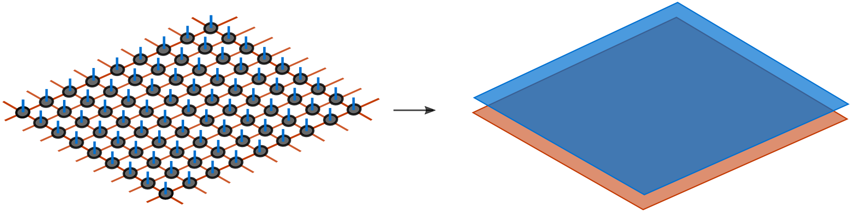


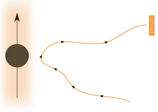
Concours chargé de recherche CNRS
Institut Henri Poincaré, Paris, France
March 28th, 2019

Alexander von Humboldt
Stiftung/Foundation

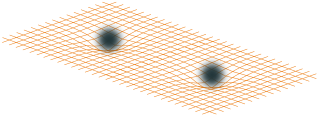
MPQ

3 questions in quantum mechanics

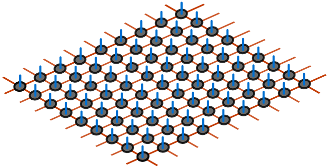
Continuous measurement
How to gently measure and control quantum systems?

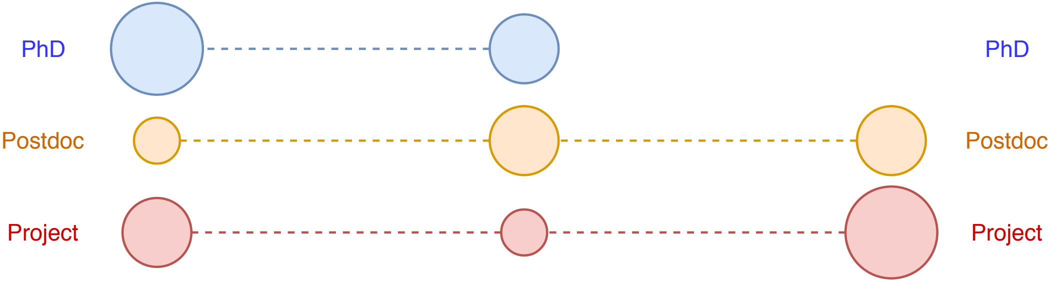


Gravity and quantum
Could gravity, in principle, not be quantum?

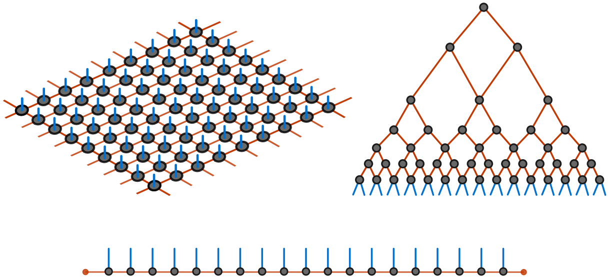


Many-body & QFT
How to efficiently parameterize many-body and QFT states





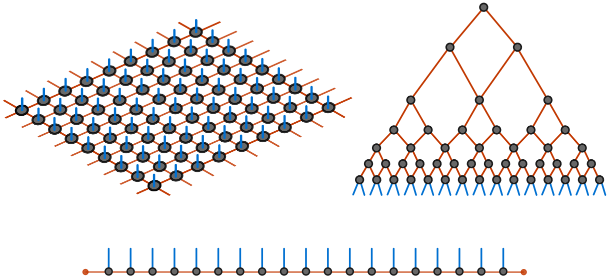
Tensor network states: a tool



Applications

- Quantum information theory
- Statistical Mechanics
- Quantum gravity
- **Many-body quantum**

Tensor network states: a tool



Applications

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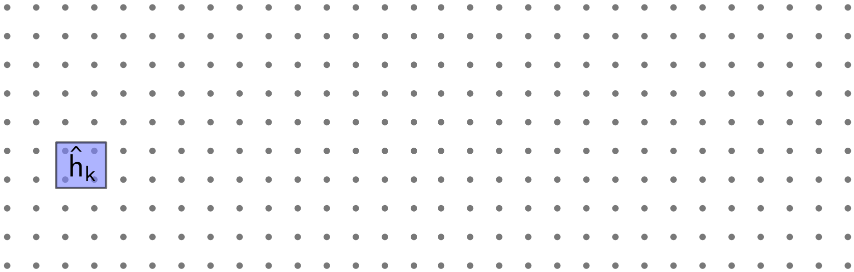
Negative theology

- **Not** covariant/geometric objects $g_{\mu\nu}$ or $R^{\sigma}_{\mu\nu\kappa}$
- **Not** tensor **models** [Rivasseau, Gurau, ...]

Many-body problem



Problem

Finding low energy states of

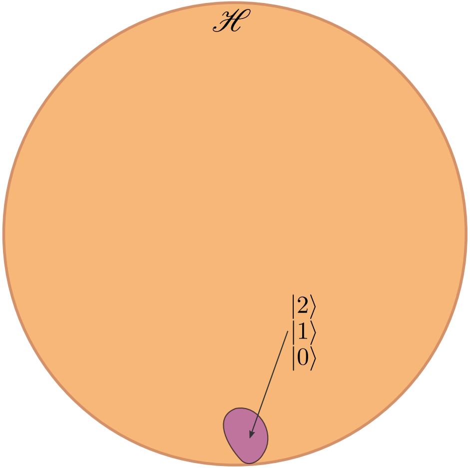
$$\hat{H} = \sum_{k=1}^{N} \hat{h}_k$$

is **hard** because $\dim \mathscr{H} \propto D^N$

Possible solutions

- Perturbation theory
- Monte Carlo
- Bootstrap IR fixed point
- **Variational optimization** (*e.g.* Mean Field, TCSA, tensor networks)

Variational optimization



Generic (spin $D/2$) state $\in \mathscr{H}$:

$$|\psi\rangle = \sum_{i_1, i_2, \cdots, i_n} c_{i_1, i_2, \cdots, i_N} |i_1, \cdots, i_N\rangle$$

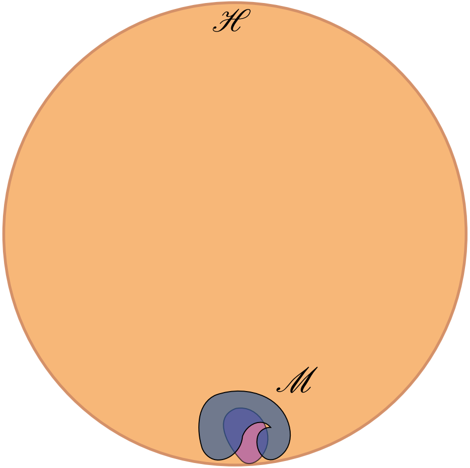
Exact variational optimization

To find the ground state:

$$|0\rangle = \min_{|\psi\rangle \in \mathscr{H}} \frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle}$$

- $\dim \mathscr{H} = D^N$

Variational optimization



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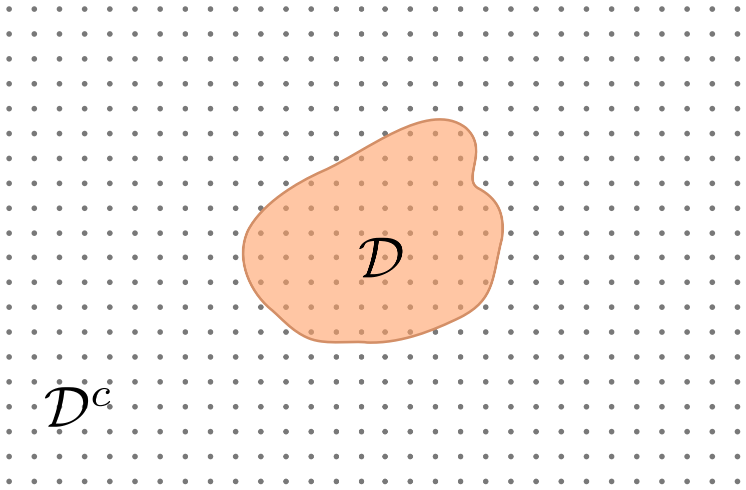
Approx. variational optimization

To find the ground state:

$$|0\rangle = \min_{|\psi\rangle \in \mathscr{M}} \frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle}$$

- $\dim \mathscr{M} \propto \mathrm{Poly}(N)$ or fixed

Interesting states are weakly entangled



Low energy state

$|\psi\rangle = |0\rangle$ or $|1\rangle$ ...

Reduced density matrix

$\rho = \mathrm{tr}_{\mathcal{D}^c}\left[|\psi\rangle\langle\psi|\right]$

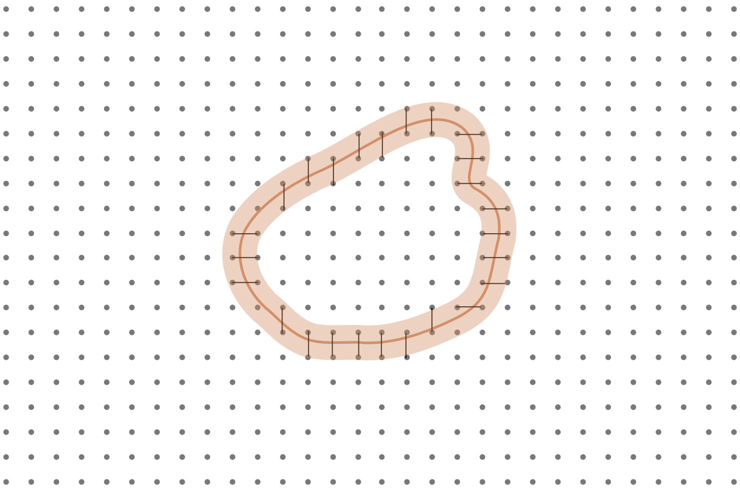
Entanglement entropy

$S = -\mathrm{tr}\left[\rho \log \rho\right]$

Area law

$S \propto |\partial \mathcal{D}|$

Interesting states are weakly entangled



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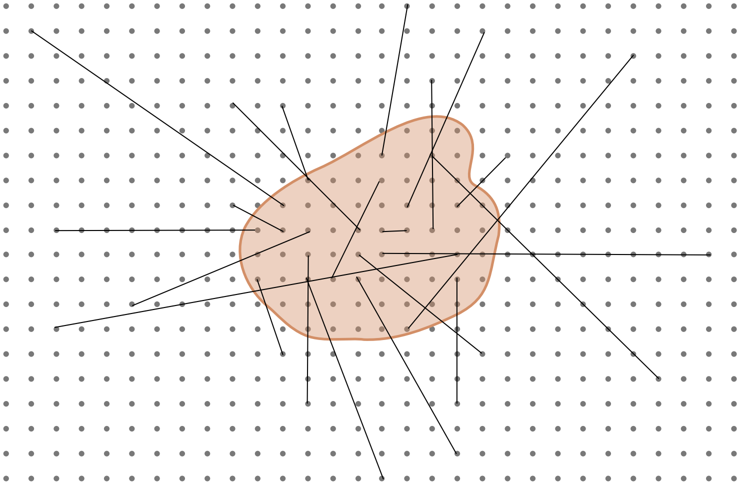
Typical states are strongly entangled



Random state

$|\psi\rangle = U_{\mathrm{Haar}}|\mathrm{trivial}\rangle$

Reduced density matrix

$\rho = \mathrm{tr}_{\mathcal{D}^c}\left[|\psi\rangle\langle\psi|\right]$

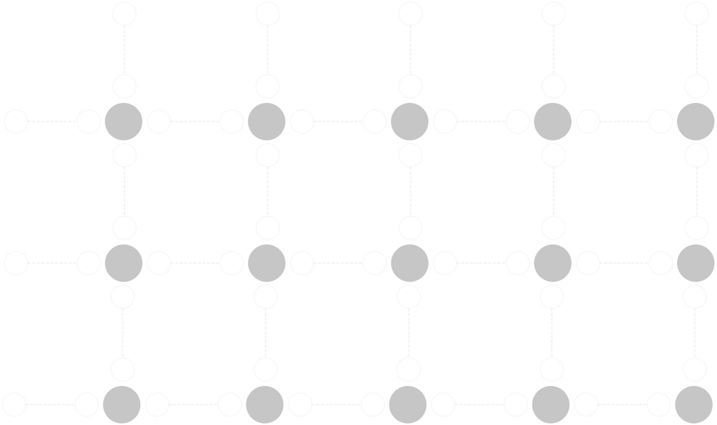
Entanglement entropy

$S = -\mathrm{tr}\left[\rho \log \rho\right]$

Volume law

$$S \propto |\mathcal{D}|$$

Constructing weakly entangled states



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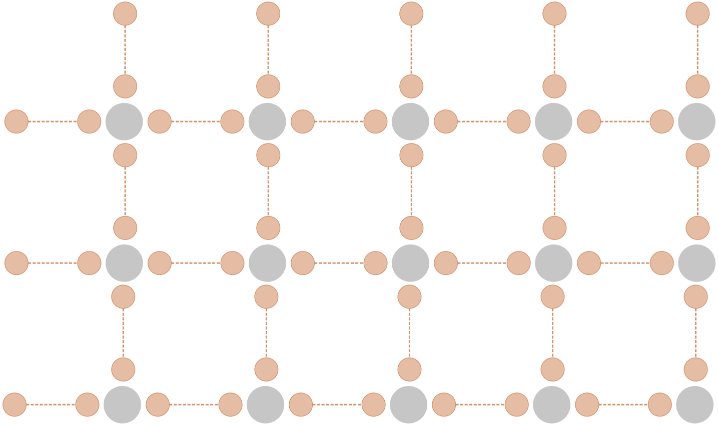
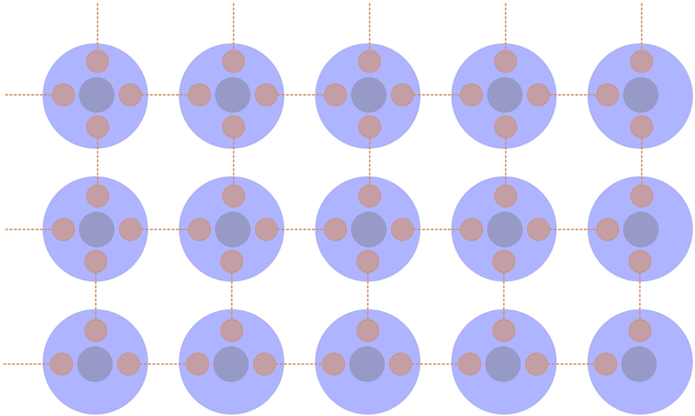


1. Put auxiliary **maximally entangled** states between sites

 $= \sum_{j=1}^{\chi} |j\rangle|j\rangle$

Constructing weakly entangled states



1. Put auxiliary **maximally entangled** states between sites

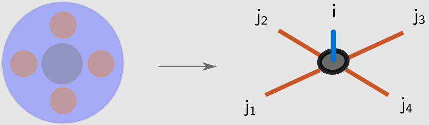
 $= \sum_{j=1}^{\chi} |j\rangle|j\rangle$

2. Map to initial Hilbert space on each site

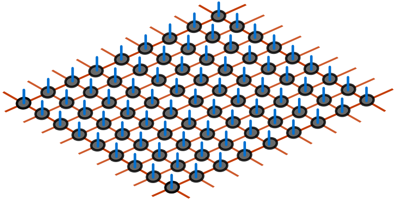
 $= A : \mathbb{C}^{4\chi} \to \mathbb{C}^D$

Tensor network states: definition

Why “tensor” network?



$$A : \mathbb{C}^{4\chi} \to \mathbb{C}^{d} \quad \longrightarrow \quad A^{i}_{j_1,j_2,j_3,j_4}$$

$|A\rangle =$ 

with tensor contractions on links

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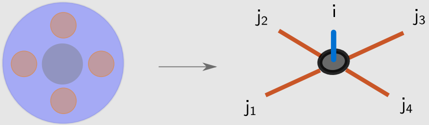

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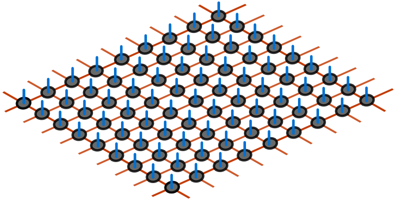
$|A\rangle =$ 

with tensor contractions on links

Optimization

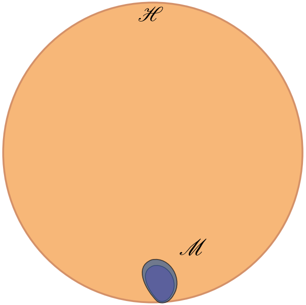
Find best A for fixed χ $\quad (D \times \chi^4$ coeff.)

$$E_0 \simeq \min_A \frac{\langle A|\hat{H}|A\rangle}{\langle A|A\rangle}$$

for example go down $\frac{\partial E}{\partial A^{i}_{j_1, j_2, j_3, j_4}}$

Some facts

$d = 1$ spatial dimension

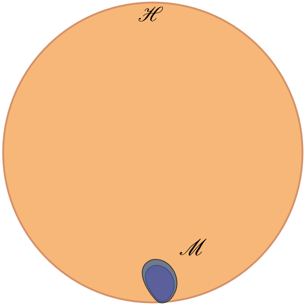


Theorems (colloquially)

1. For gapped H, tensor network states $|A\rangle$ approximate well $|0\rangle$ with χ fixed
2. **All** $|A\rangle$ are ground states of gapped H

Some facts

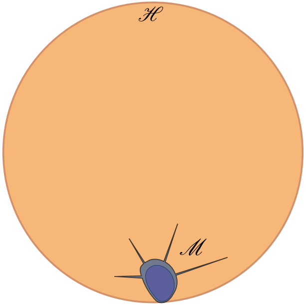
$d = 1$ spatial dimension



Theorems (colloquially)

1. For gapped H, tensor network states $|A\rangle$ approximate well $|0\rangle$ with χ fixed
2. **All** $|A\rangle$ are ground states of gapped H

$d \geqslant 2$ spatial dimension



Folklore

1. For gapped H, tensor network states $|A\rangle$ approximate well $|0\rangle$ with χ fixed
2. **Most** $|A\rangle$ are ground states of gapped H

Limitations

Hard to contract in $d \geqslant 2$

In $d \geqslant 2$ one can have:

- $|A\rangle$ known
- $\langle A|\hat{\mathcal{O}}_i\hat{\mathcal{O}}_j|A\rangle$ hard to compute exactly

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Expressive but opaque

Generally hard to interpret

- Tensor carries IR-irrelevant information
- Hard to constrain long distance behavior

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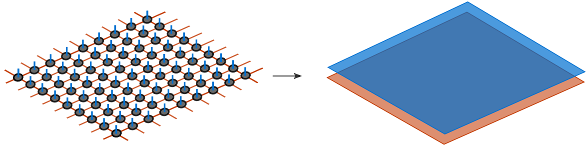
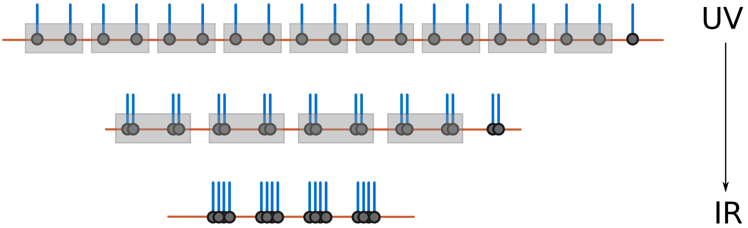
$\Longrightarrow$ Go to the continuum and **QFT**: Major objective and challenge



Continuous Matrix Product states

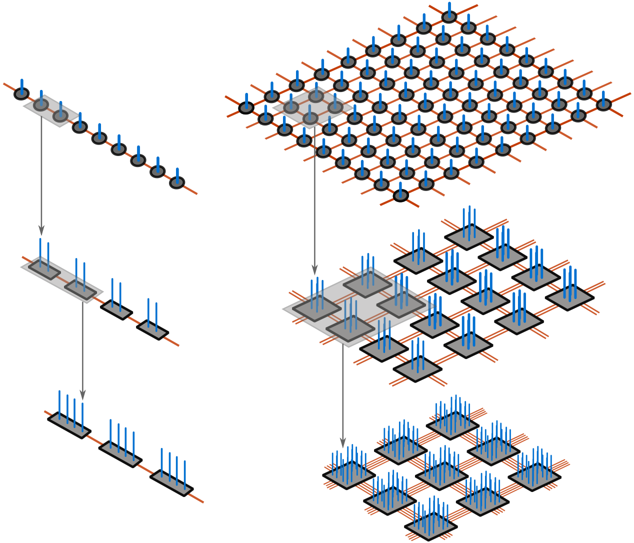
[Verstraete & Cirac 2010]: continuum limit of **M**atrix **P**roduct **S**tates ($d = 1$ tensor networks)



Works for Lieb-Liniger model (boson with contact interactions), ϕ^4, etc.
Best method on the market for $1 + 1$ QFT

But no version for $d + 1$ QFT, even "no-go" theorems

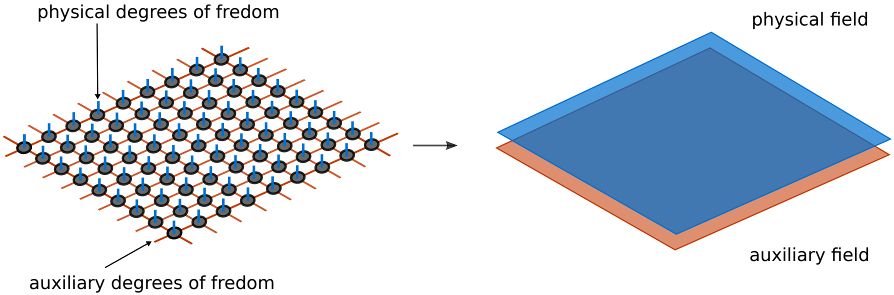
Continuous Tensor Networks: blocking



Upon **blocking**:

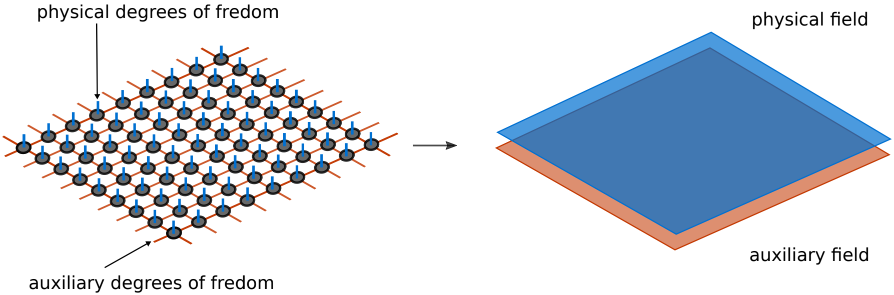
- The **physical** Hilbert space dimension D increases
- The **bond** (auxiliary space) dimension χ increases too

Result



AT, J. I. Cirac, *Phys. Rev. X 2019 (in print)*

Result



AT, J. I. Cirac, *Phys. Rev. X 2019 (in print)*

Continuous tensor network state (heuristically)

State $|\alpha\rangle$ of $d+1$ QFT from an auxiliary d dimensional theory of random fields ϕ:

$$|\alpha\rangle = \int \mathcal{D}\phi \, \exp\left\{ -\int \mathrm{d}^d x \, \mathcal{L}[\phi(x)] - \alpha[\phi(x)] \underbrace{\hat{\psi}^\dagger(x)}_{\text{creation}} \right\} |\Omega\rangle$$

1. Genuine continuum limit of discrete tensor networks
2. The toolbox is translated to the continuum

Future

Reopens the field after 8 years of only $d = 1$
So far, success **expected** from success in the discrete and continuous $d = 1$

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New **non-perturbative** method, how will it fare?

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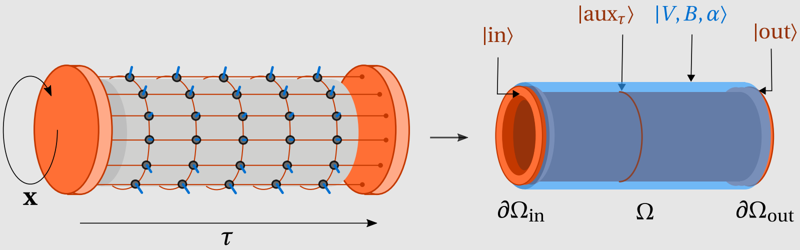
So far, success **expected** from success in the discrete and continuous $d = 1$

New **non-perturbative** method, how will it fare?

Continuous tensor network states (cTNS) for dimensional reduction

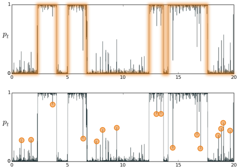


Contracting a cTNS in $2d$ $=$ Solving χ field theories in $1d$ $=$ Optimizing χ cTNS in $1d$

One can trade a dimension for a variational optimization

Summary: 3 fields, 3 main results

Quantum measurement

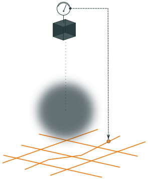


Mathematical understanding of stochastic dynamics to help control quantum systems in the lab

Main results:

- Quantum jumps
- Spikes

Gravity

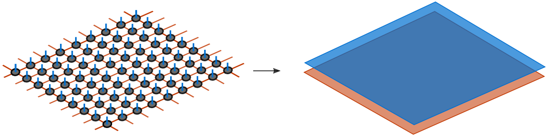


Understand if gravity could be **not** quantized

M. Results:

- Toy models

Tensor networks for QFT



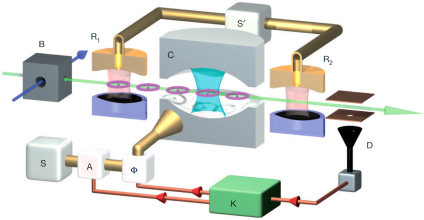
Extend a powerful **variational method** from the lattice to the continuum

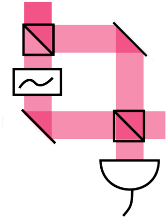
Main results:

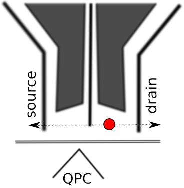
- An ansatz of continuous tensor network state
- Promising non-perturbative methods for QFT

Observation: Continuous quantum measurement and control

Experimentally:
Ubiquitous

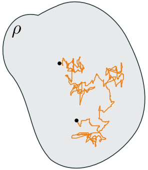






Theoretically:
Non-linear stochastic modifications of the Schrödinger equation

$$d|\psi_t\rangle = \left[-iH\,dt + \sqrt{\gamma}(A - \langle A\rangle)dW_t - \frac{\gamma}{2}(A - \langle A\rangle)^2\,dt\right]|\psi_t\rangle$$



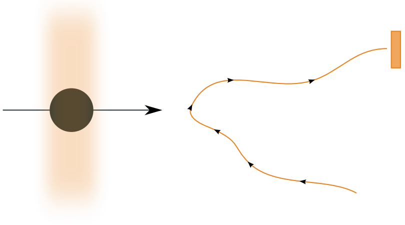
Hilbert space

Observation: Continuous quantum measurement and control

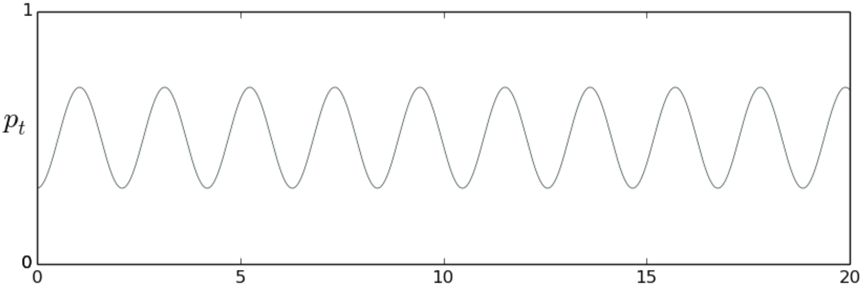
Prototypical study

Qubit in a magnetic field $\perp$ measurement basis

- $H \propto X$
- Measurement $\propto Z$
- $p_t = |\langle \psi_t | \uparrow \rangle_z|^2$



No measurement



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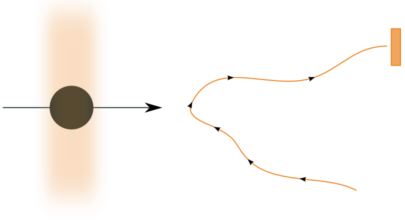
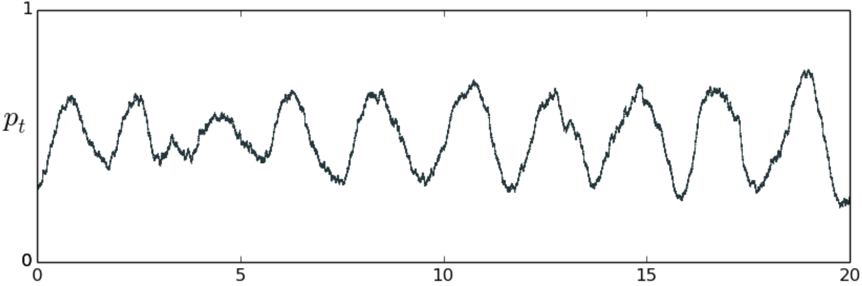


Weak measurement



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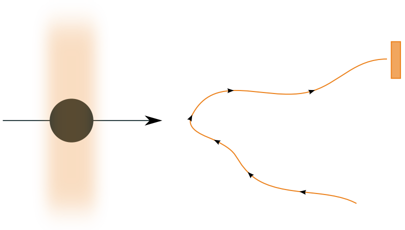
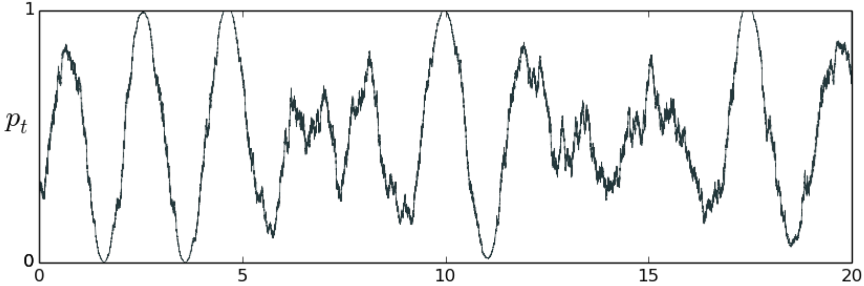


Average measurement



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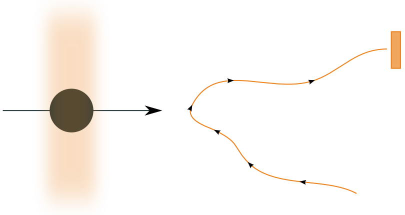
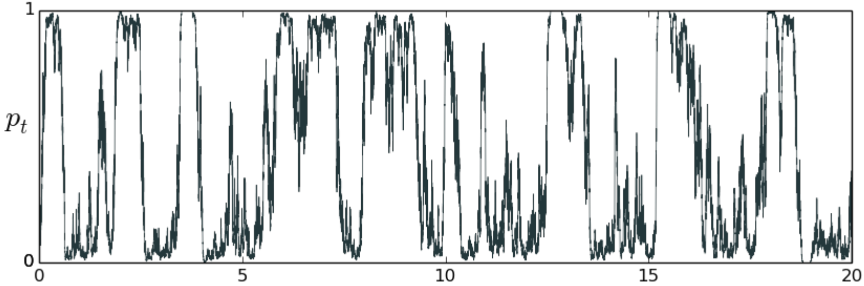


Strong measurement



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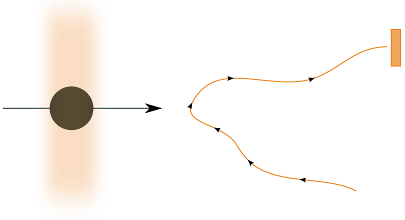
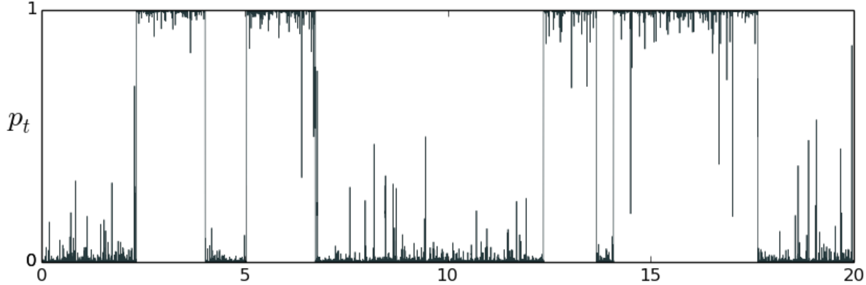


Very strong measurement



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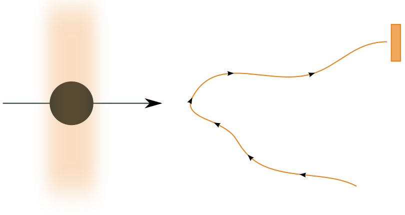
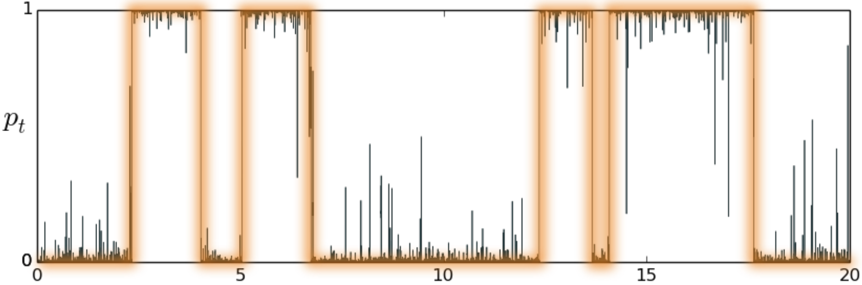


Very strong measurement

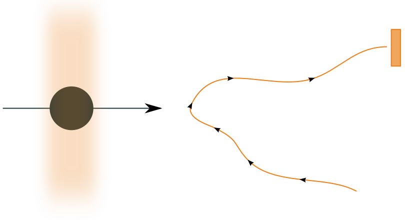


Observation: Continuous quantum measurement and control

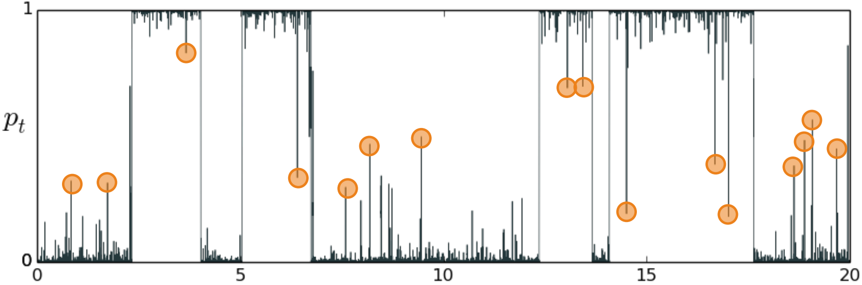
Prototypical study

Qubit in a magnetic field ⊥ measurement basis

- $H \propto X$
- Measurement $\propto Z$
- $p_t = |\langle\psi_t|\uparrow\rangle_z|^2$



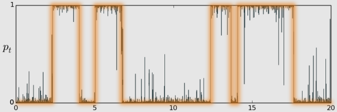
Very strong measurement



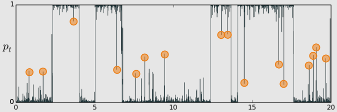
Results

Strong continuous measurement

1. Jump theorem



2. Spike theorem



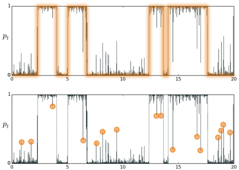
- ◇ M Bauer, D Bernard, AT JPA 2015
- ◇ AT, M Bauer, D Bernard PRA 2015
- ◇ M Bauer, D Bernard, AT JPA 2016

Others

1. Control
   - ◇ A T, M Bauer, D Bernard EPL 2014
2. Optimal measurement
   - ◇ AT, PRA 2016
3. Exact results
   - ◇ AT, PRA-Rapid 2018
4. Non-Markovian exploration
   - ◇ AT, Quantum 2017
5. Many-body exploration
   - ◇ X Cao, AT, A De Luca, 2018

Summary: 3 fields, 3 main results

Quantum measurement

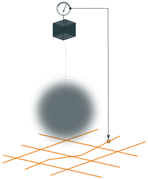


Mathematical understanding of stochastic dynamics to help control quantum systems in the lab

Main results:

- Quantum jumps
- Spikes

Gravity

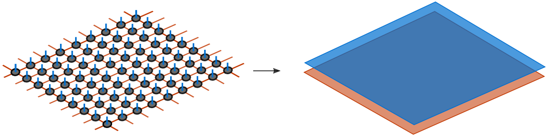


Understand if gravity could be **not** quantized

M. Results:

- Toy models

Tensor networks for QFT



Extend a powerful **variational method** from the lattice to the continuum

Main results:

- An ansatz of continuous tensor network state
- Promising non-perturbative methods for QFT