Some open problems I think about

at work, in the shower, or while running

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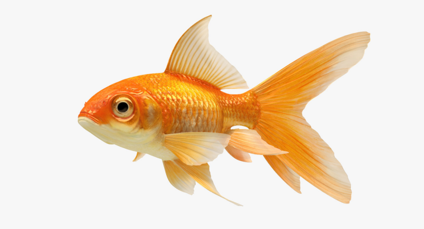


MPQ Theory division workshop
Somewhere in Germany
October 22nd, 2019

Alexander von Humboldt
Stiftung/Foundation

MPQ

List of problems

1 – [tensor networks] Is there a good continuous MERA?

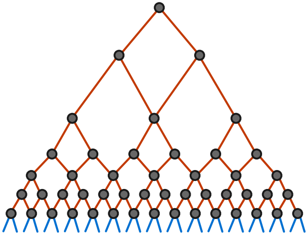
2 – [entanglement] Is there a simple model of measurement induced phase transition?

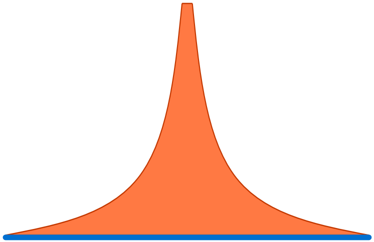
3 – [foundations] Can relativistic quantum field theories be made open?

4 – [real world] Should actinides be burnt or burried?

5 – [random] A list of other open problems

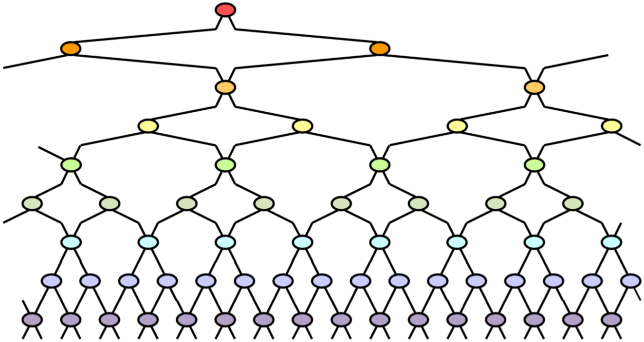
Q1: Is there a good continuous MERA?

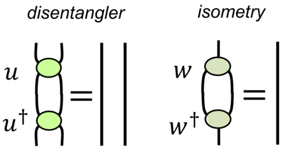




What is the MERA?

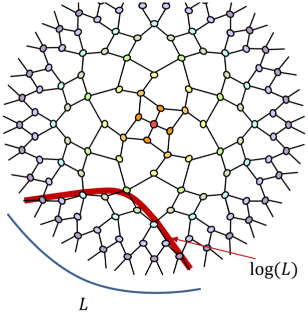
The **Multiscale Entanglement Renormalization Ansatz** is a tensor network with a particular structure adapted to **critical systems** [courtesy of Guifre Vidal]



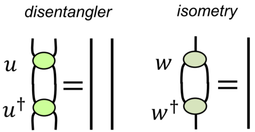


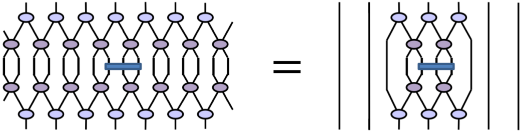
Some facts about the MERA

1 – Because of **geometry**: $S \propto \log L$



2 – Because of **special tensors**: renormalization is **local** – "strict causal cone"





Why would the cMERA be nice?

- Apply directly to QFT without discretization
- Renormalize continuously (and not by factors of 2)

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- Apply directly to QFT without discretization
- Renormalize continuously (and not by factors of 2)
- Translation invariant even without Lorenzo's galactic brain

cMERA proposals – original Haegeman *et al.* 2011

Start from $\mathscr{H}$ of non-relativistic QFT: $[\psi(x), \psi^\dagger(y)] = \delta(x-y)$
and define:

$$|\Psi\rangle = U|\Omega\rangle = \mathcal{P}\exp\left(\int_{s_{UV}}^{s_{IR}} L + K\right)|\Omega\rangle$$

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- Equivalent of the "coarse grainer" (3 legged tensor) $\psi(x) \to \psi(e^{-s}x)$

$$L := -\frac{i}{2}\int \psi^\dagger(x)x\frac{d\psi(x)}{dx} - x\frac{d\psi^\dagger(x)}{dx}\psi(x)dx \tag{1}$$

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- Equivalent of the disentangler (4 legged tensor)

$$K = \int \mathrm{d}x\mathrm{d}y\, K_2(x,y)\,\psi^{(\dagger)}(x)\psi^{(\dagger)}(y) + \int K_3\psi\psi\psi + \cdots$$

where $K_j(x_1, \cdots, x_j)$ is $\sim \Lambda^{-1}$ local, i.e. $|K_j| \ll 1$ if $|x_k - x_i| \gg \Lambda^{-1}$

cMERA proposals – perturbative refinements

Remember definition:

$$|\Psi\rangle = U|\Omega\rangle = \mathcal{P}\exp\left(\int_{s_{UV}}^{s_{IR}} L + K\right)|\Omega\rangle$$

K free, but $|\psi\rangle$ computable for K at most quadratic – "**Gaussian cMERA**"

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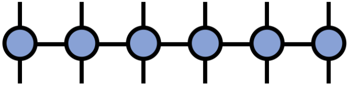
$\Rightarrow$ do perturbation theory for weakly interacting fields [Cotler et al. 2017]

Nice that it can be done, but probably not interesting fixed points.

cMERA proposals – “magic” entangler

Take K a continuous matrix product operator [Zou, Ganahl, Vidal 2019]:

$$K = \mathrm{tr}_{\mathrm{aux}} \left\{ \mathcal{P} \exp \left[\int \mathrm{d}x \; Q \otimes \mathbb{1} + R \otimes \psi(x) + \bar{R} \otimes \psi^{\dagger}(x) \right] \right\}$$



1. breaks locality
2. preserves the geometry
3. two matrices parameterize everything

cMERA questions

1. For a “magic” cMERA with R and Q fixed, what is the conformal data?

$$\Delta, \mathcal{O}_\Delta(x), C_{ijk} = f(R, Q)?$$

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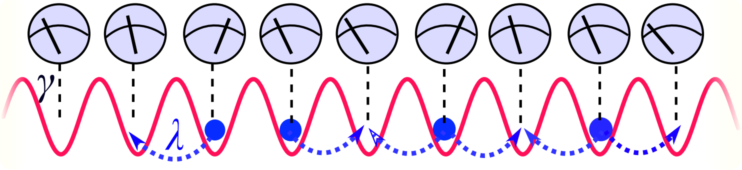
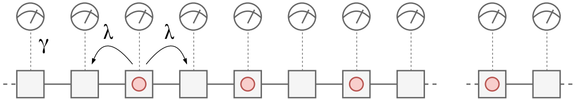
3. Can a cMERA be contracted by $\simeq$ TCSA
[truncation of the field algebra + exact diag.]

Q2: Is there a simple model of measurement induced entanglement phase transition?



Continuously measured free fermions



Consider **free fermions** on a line:

$$H = \lambda \sum_{j=1}^{N} a_j^\dagger a_{j+1} + a_{j+1}^\dagger a_j$$

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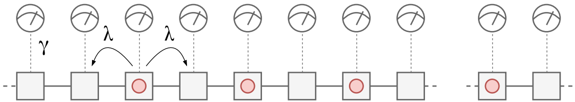

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add continuous measurement of number n_j on each site:

$$\mathrm{d}\,|\psi_t\rangle = -iH\mathrm{d}t\,|\psi_t\rangle + \sum_{i=1}^{L} \left(\sqrt{\gamma}\,[n_i - \langle n_i \rangle_t]\,\underbrace{\mathrm{d}W_t^i}_{\text{noise}} - \frac{\gamma}{2}\,[n_i - \langle n_i \rangle_t]^2\,\mathrm{d}t \right) |\psi_t\rangle$$

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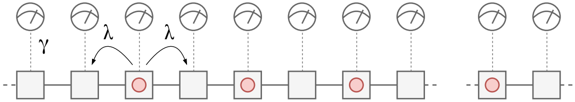

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Gaussianity is preserved $\rightarrow$ non-linear closed equation for $D_{ij} = \langle a_i^\dagger a_j\rangle$

Continuously measured free fermions

Basic non-linear continuous measurement evolution:

$$\mathrm{d}\,|\psi_t\rangle_{\text{meas.}} = -iH\mathrm{d}t\,|\psi_t\rangle + \sum_{i=1}^{L}\left(\sqrt{\gamma}\,[n_i - \langle n_i\rangle_t]\,\mathrm{d}W_t^i - \frac{\gamma}{2}\,[n_i - \langle n_i\rangle_t]^2\,\mathrm{d}t\right)|\psi_t\rangle$$

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Upon measurement randomness averaging, we would get the **Lindblad equation**:

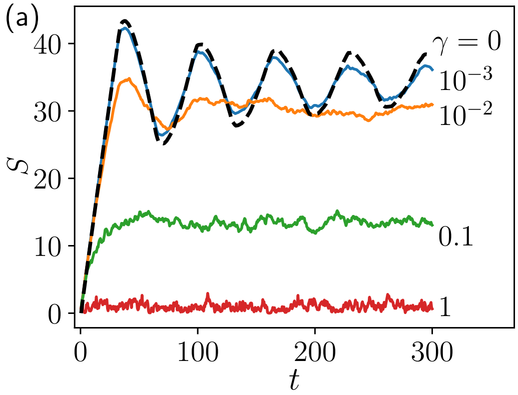
$$\partial_t\rho_t = -i[H,\rho] - \frac{\gamma}{2}\sum_j [n_j,[n_j,\rho_t]]$$

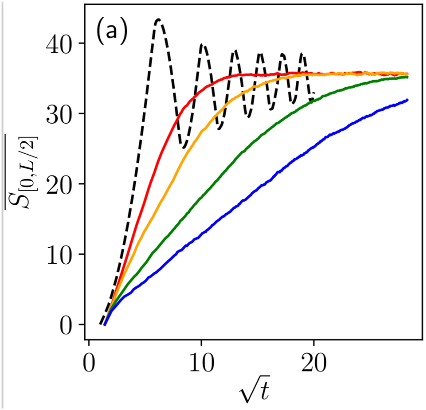
Note that pure unitary noise would give **same** Lindblad:

$$\mathrm{d}|\psi_t\rangle_{\text{noise}} = -iH\mathrm{d}t|\psi_t\rangle - i\sum_j n_j\,\mathrm{d}W_t^i|\psi_t\rangle = -iH_{\text{noise}}|\psi_t\rangle$$

Results

Now quench and look at entanglement entropy:





Entanglement varies wildly depending on "unraveling" after a quench:

1. $|\psi_t\rangle_{\text{meas.}}$ has **area** law entanglement for all γ
2. $|\psi_t\rangle_{\text{noise}}$ has **volume** law entanglement for all γ

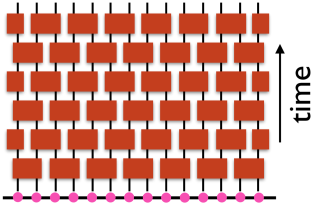
$\rightarrow$ no phase transition, very good generalized hydrodynamics

Duck shooting intuition

A quench,

- creates lots of entangled excitation
- they propagate at constant speed
- they get randomly killed by measurement

BUT

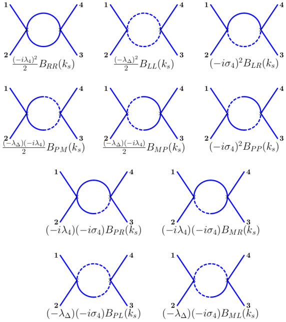


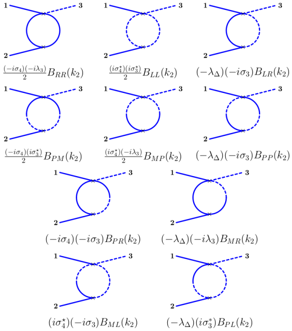
random circuits show there can be a phase transition in both cases for γ_c

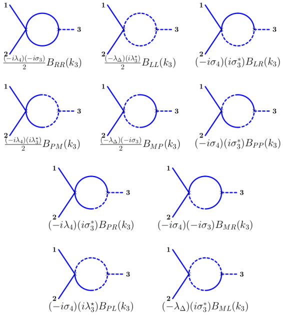
Some weird scrambling thingy must be going on (ducks turn into snakes)

Is there a simple model, physically reasonable, numerically manageable, that shows both phase transitions?

Q3: Can relativistic quantum field theories be made open?







Markovian open system dynamics in quantum mechanics

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_t = -i[H, \rho_t] + \sum_{k=1}^{n} A_k \rho A_k^\dagger - \left\{ A_k^\dagger A_k, \rho_t \right\}$$

Origins:

1. Interaction with bath in Markovian limit
2. Repeated interaction with discrete ancillas

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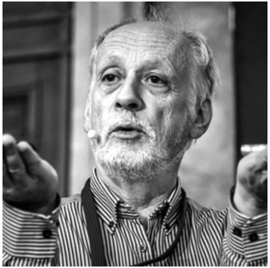
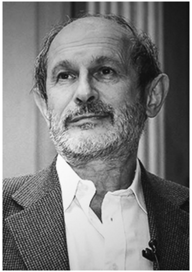
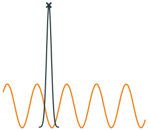
Origins:

1. Interaction with bath in Markovian limit
2. Repeated interaction with discrete ancillas
3. What if it is fundamental?

Dynamical reduction program

In the 80's people proposed to add a fundamental collapse of the wavefunction in the Schrödinger equation:

so Lindblad dynamics fundamental $\simeq$ fundamental collapse of $|\psi\rangle$

BUT non-relativistic

Field theory

It is hard!

Difficulties:

- Vacuum unstable, decays into particle pairs
- Infinite energy density increase $\partial_t \mathrm{tr}[h(x)\rho_t] = +\infty$
- S-matrix picture blows up
- No LSZ reduction formulas

But:

- Free models seem to make sense regardless of infinite energy
- Interacting models look formally renormalizable
- So maybe the infinites are spurious

What has been done

Preskill's notes (35 pages) [no hyperlink, but indexed by Google]

Diagrammatics for the field theoretic Lindblad equation

Note Title 7/24/2014

Feynman diagrams are derived from a "superaction" of the form

$$\exp(iS - iS' + \$)$$

where S acts on ket side and S' on bra side of a density operator

The $\$$ term acts on both. It can be a sum of terms of the form $(\mathcal{O} - \mathcal{O}')^2$ where $\mathcal{O}$ is a function of fields (but not derivatives). This corresponds to CP Lindblad term $2\mathcal{O}\rho\mathcal{O} - \mathcal{O}^2\rho - \rho\mathcal{O}^2$

The simplest Lorentz invariant superterms are $(\phi - \phi')^2$ and $(\phi^2 - \phi'^2)^2$

Let's compute some loop corrections -- Are such theories renormalizable? That is -- can divergences be cancelled by counterterms of the same form?

In the Feynman rules:

1) Normal vertices are imaginary but supervertices are real.
2) The propagators for bra internal lines and ket internal lines have opposite sign.
3) Bra and ket propagators have opposite i epsilon prescriptions.

Let's consider the theory

$$S = \tfrac{1}{2}(\partial_\mu\phi)^2 - \tfrac{1}{2}m^2\phi^2 \quad \text{and} \quad \$ = \tfrac{\lambda}{4!}(\phi^2 - \phi'^2)^2$$

we'll use solid lines to denote kets and dotted lines for bras.

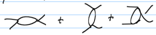
As a warmup, recall $\mathcal{L} = \frac{1}{4!}\lambda\phi^4$

In one loop  $= (-i\lambda)^2 \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\left(\frac{i}{k^2 - m^2 + i\epsilon}\right)^2$ $= \frac{\lambda^2}{32\pi^2}\int dk_E^2\, k_E^2 (i) \frac{1}{k_E^4} = \frac{i\lambda^2}{32\pi^2}\ln\Lambda^2$

symmetry factor

wick rotation

There are 3 such diagrams:



An incoming particle can be paired at a vertex with any of 3 other incoming particles

 $-i(\delta\lambda)$ Cancel divergence with a counterterm

$$\Rightarrow \delta\lambda = \frac{3}{32\pi^2}\lambda^2 \ln\Lambda^2$$

If we define renormalized coupling at scale μ, bare coupling is

$$\lambda_0 = \lambda_\mu + \frac{3}{16\pi^2}\lambda_\mu^2 \ln\frac{\Lambda}{\mu}$$

we keep λ_0 fixed as μ changes: $\mu\frac{d}{d\mu}\lambda_0 = 0$

$$\Rightarrow \mu\frac{d}{d\mu}\lambda_\mu = \frac{3}{16\pi^2}\lambda_\mu^2$$

Not asymptotically free

As another warm up, to check diagram combinatorics, consider $SO(2)$ invariant theory

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu\phi_1^2 + \tfrac{1}{2}\partial_\mu\phi_2^2 - \tfrac{1}{2}m^2(\phi_1^2 + \phi_2^2) - \tfrac{\lambda}{4!}(\phi_1^2 + \phi_2^2)^2$$

The counterterm should be $SO(2)$ invariant

What has been done

Preskill's notes (35 pages) [no hyperlink, but indexed by Google]

going on.

In that case the term in the dissipator with numerator ~ omega^2 comes from gamma_p, and the constant term

E.g. $\omega = -\omega_0 \Rightarrow 2\pi f(\omega) = \frac{1}{m\omega_0} \frac{\gamma_p \omega^2 + \gamma_x \omega_0^2}{(\omega - \omega_0)^2} e^{i\omega t}$

$$\Rightarrow 2\pi i f'(-\omega_0) = \frac{i}{m\omega_0}\left(\frac{-\gamma_p}{2\omega_0} + \frac{2(\gamma_p+\gamma_x)}{8\omega_0} + \frac{\gamma_p+\gamma_x}{4}(it)\right) e^{-i\omega_0 t}$$

$$= \frac{i}{m\omega_0}\left(\frac{\gamma_p-\gamma_x}{4\omega_0} + \frac{\gamma_p+\gamma_x}{4}(it)\right) e^{-i\omega_0 t}$$

$t > 0$

$$G(t) = \frac{e^{-i\omega_0 t}}{2m\omega_0}\left(\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} + \left[\frac{i(\gamma_p-\gamma_x)}{2\omega_0} - \frac{1}{2}(\gamma_p+\gamma_x)t\right]\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\right)$$

$t < 0$

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(Same function, but with t replaced by -t.)

Wow ... what does *that* mean?

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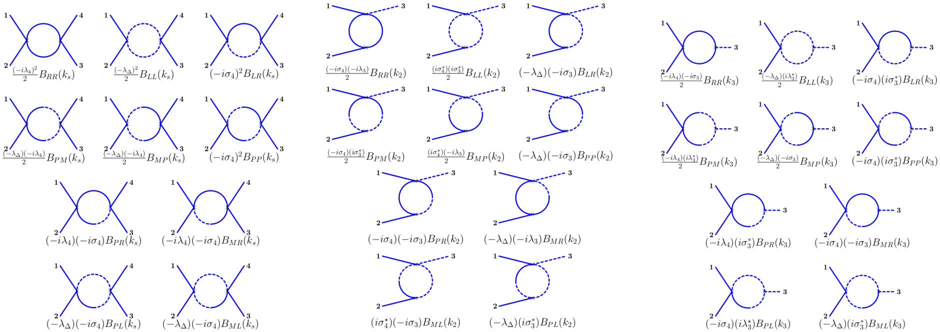
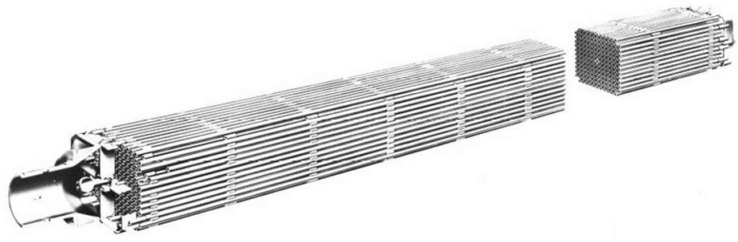
What has been done

Some brave bois computed 1-loop β-function with fingers crossed
[Avinash Baidya, Chandan Jana, R. Loganayagam, and Arnab Rudra]



Q4: Should actinides be burnt or burried?



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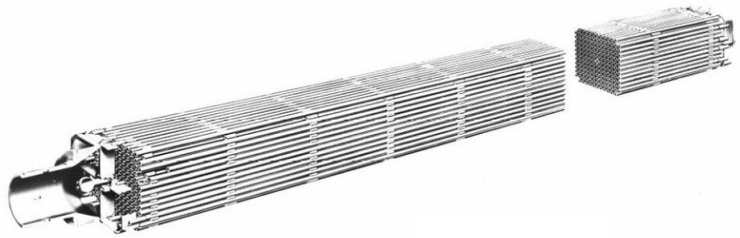
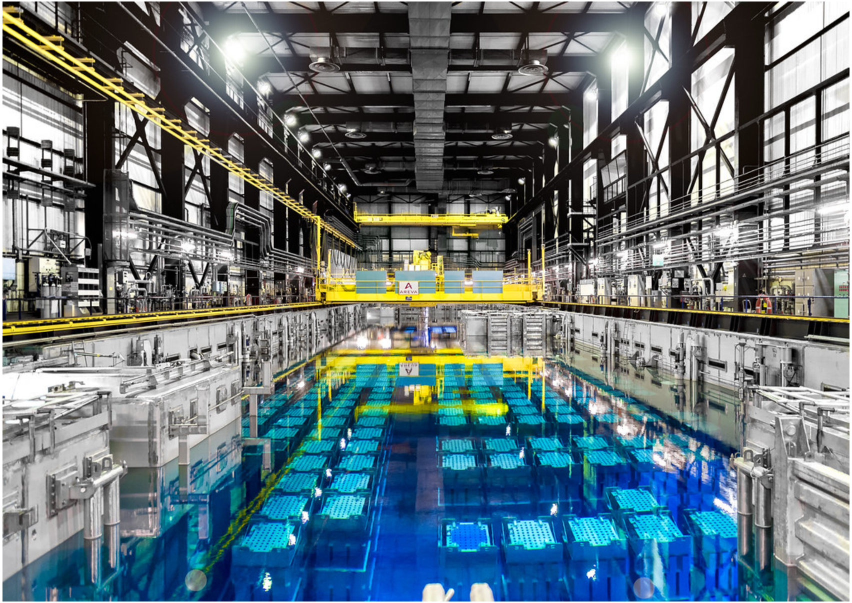
Composition of spent nuclear fuel



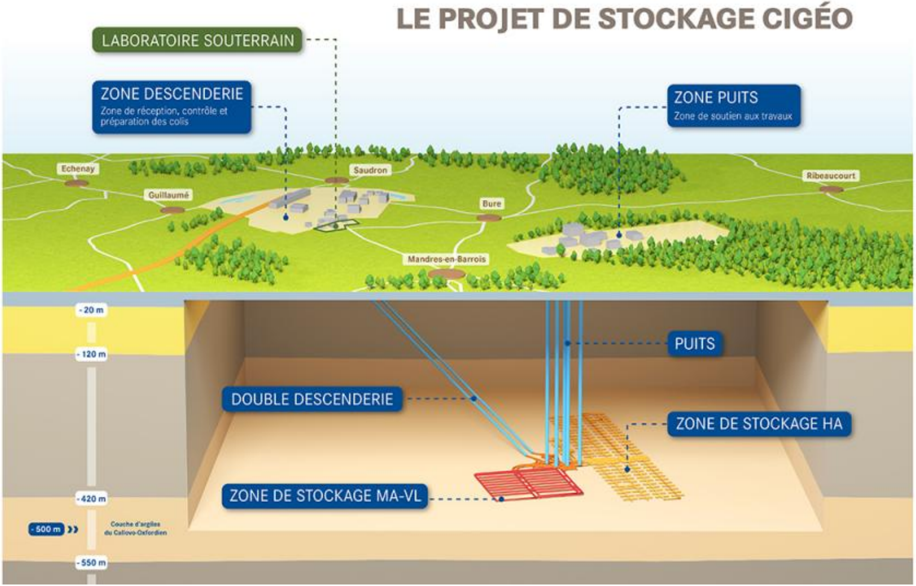
For French pressurized water reactors, with 4% enriched fuel, and standard burnup:

1. $\simeq$ 95% uranium (at 0.9% of ^{235}U)
2. $\simeq$ 4% fission products (some stable, but also ^{137}Cs, ^{129}I, ^{90}Sr,...)
3. $\simeq$ 1% plutonium ($\simeq$ 60% of ^{239}Pu)
4. $\simeq$ 0.1% minor actinides (^{241}Am, Np, Cm, ...)

Where they are temporarily



Deep geological storage



Back to the fuel

1. 95% uranium (at 0.9% of ^{235}U) → eternal but weakly radioactive
2. 4% fission products → very radioactive, but short lived (half-life ≃ 30 y)
3. 1% plutonium → very radioactive, long lived (half-life 24000 y)
4. 0.1% minor actinides → very radioactive, quite long lived (half-life ≃ 500 y)

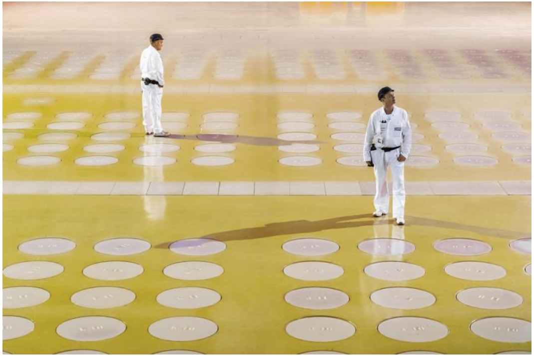
Standard fuel reprocessing and plutonium reuse

At first order of approximation $^{239}\text{Pu} \simeq\ ^{235}\text{U}$
$\rightarrow$ blend it with ^{238}U at roughly 8% $\rightarrow$ boom! new fuel to burn

[10% of French electricity from such “recycled” fuel]

Where the rest goes

Minor actinides + fission products are **vitrified**



Could we do the same for americium?

Americium is less fissile than plutonium, not enough to maintain reaction

Yes but no in current reactors:

- Light water reactor "can" consume americium
- Produces small amounts of berkelium and californium, with energetic γ

Could we do the same for americium?

Americium is less fissile than plutonium, not enough to maintain reaction

Yes but no in current reactors:

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- Produces small amounts of berkelium and californium, with energetic γ

Yes in better reactors:

- Burnable at a slow rate in **fast neutron reactors**
- Burnable slightly faster in **accelerator driven subcritical reactors**

Some reactors that can burn americium

Phenix, Marcoule, France (retired)

BN800, Beloyarsk, Russia

Myrrha, Belgium (project)

Should we do the same for americium?

Unclear ...

CEA ($\simeq$ French national labs) showed:

- americium can be efficiently separated from spent fuel
- fast neutron reactors can burn it progressively in $\simeq$ 100 years **if** $n_{\text{fast}} \geqslant n_{\text{total}}/2$

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IRSN ($\simeq$ national expert on nuclear safety and radiation protection)

- Increases by a factor of 10 the amount of americium in processing plants and reactors
- Fuel massively radioactive and very hard to manage
- Overall much higher risks than just putting it underground as it is not soluble

What people think

A sizeable fraction of the public thinks that:

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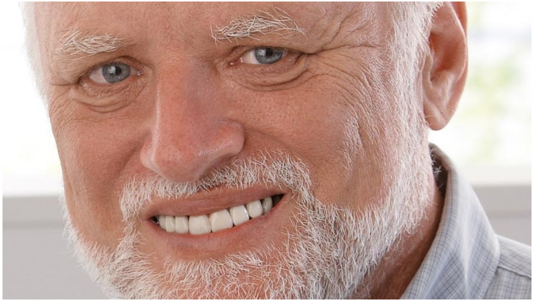

Random open-problems

- Can conformal bootstrap methods be used to make tensor networks more efficient?
- Is there a meaningful functional renormalization group for tensor networks?
- How can the lattice field theory tensor network renormalization be applied to interacting fields
- Can one use neural networks as ansatz for generating functionals $Z[j]$ instead of states $|\psi\rangle$
- What is the best way to falsify collapse models
- What is the best way to find signatures that gravity is classical or quantum?
- Can one combine Gaussian variational optimization and truncated Hilbert space approaches (exact diagonalization) in QFT?