Spontaneous Symmetry Breaking

Lecture 0: introduction to the introduction

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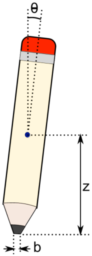


Mock-up lecture, Universität Leipzig
December 18th, 2020

Symmetries go from causes to effects

Lorsque certaines causes produisent certains effets, les éléments de symétrie des causes doivent se retrouver dans les effets produits.

Lorsque certains effets révèlent une certaine dissymétrie, cette dissymétrie doit se retrouver dans les causes qui lui ont donné naissance.

Curie's principle (1894)

When causes produce effects, the symmetries of the causes have to appear in the effects.
When effects have some asymmetry, the asymmetry is to be found in the causes.

Causes = laws (dynamical equations) + initial conditions
Effects = solutions

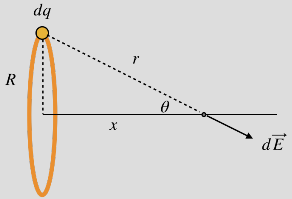
Pierre Curie
Nobel Prize 1903
(for radioactivity)

Applications, electromagnetism, polarization...

SUR LA SYMÉTRIE DANS LES PHÉNOMÈNES PHYSIQUES, SYMÉTRIE D'UN CHAMP ÉLECTRIQUE ET D'UN CHAMP MAGNÉTIQUE;

PAR M. P. CURIE.

Electric fields



- If the charge distribution ρ is symmetric by rotation along x, $\vec{E}$ should too

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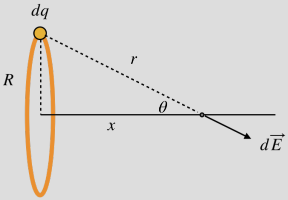

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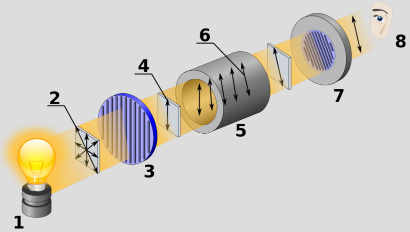
Optical rotation



- A material that rotates the polarization of light has to be **chiral**

Converse?

La réciproque de ces deux propositions n'est pas vraie, au moins pratiquement, c'est-à-dire que les effets produits peuvent être plus symétriques que les causes. Certaines causes de dissymétrie peuvent ne pas avoir d'action sur certains phénomènes ou du moins avoir une action trop faible pour être appréciée, ce qui revient pratiquement au même que si l'action n'existait pas.

No converse (1894)

The effects can be **more** symmetric than the causes.

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Symmetry breaking in classical physics

A symmetry can be unstable, and thus **spontaneously** broken:

1. The space of all stable solutions still has the symmetry
2. A single solution does not

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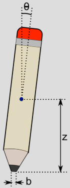

- Continuously many stable positions, $SO(2)$ broken

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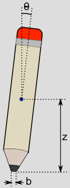

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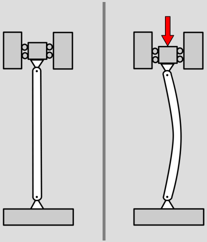
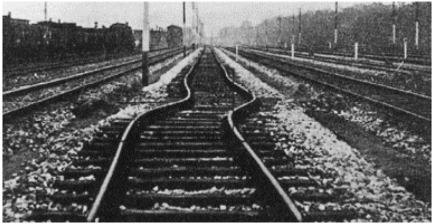
Buckling



- 2 stable positions, $\mathbb{Z}_2$ broken

Why old trains do “taktak taktak taktak”



Buckling can happen when rails get too hot

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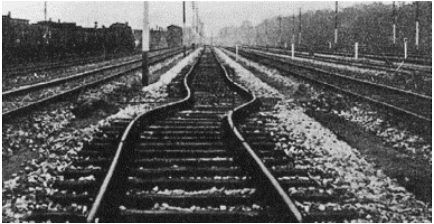

Buckling can happen when rails get too hot

Fishplates used to allow the rail to dilate (now soldered + strong lateral anchor to avoid SSB)

Symmetry breaking in statistical mechanics

Main idea: fluctuations can restore the symmetry, prevent symmetry breaking

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Toy model of magnetism: Ising model

$S_i = \pm 1$ and $Z_\beta = \sum \mathrm{e}^{-\beta E}$

$$E = -\sum_{\langle i,j \rangle} S_i S_j$$

$\langle i, j \rangle$ means i and j nearest neighbors

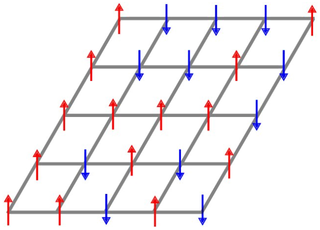


image from Sasha Wald's thesis

Intuition:

1. Minimizing energy tends to align the spins
2. Maximizing entropy forces the visit of the more common randomly aligned configurations

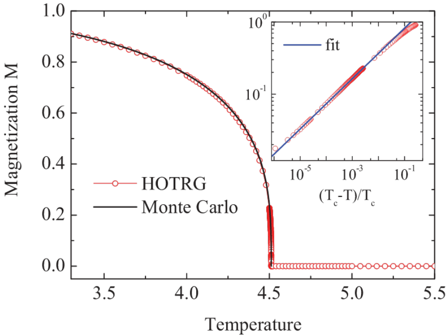
Ising model facts

- In $d = 1$, no symmetry breaking **entropy always wins** (Ising, 1924)
- In $d = 2$, symmetry breaking at $T_c = 2/\log(1+\sqrt{2})$ (Kramers and Wannier, 1941)
- In $d = 3$, symmetry breaking at $T_c \simeq 4.51154(4)$

Magnetization in $d = 3$

A good **order parameter**

$$M = \lim_{N\to+\infty} \frac{1}{N} \sum_{i=1}^{N} \langle S_i \rangle$$



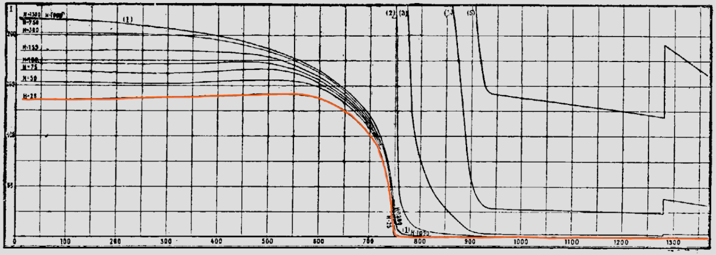
Xie et al., `arXiv:1201.1144`

Application to ferromagnets

Observation by Curie (1895)

A magnet that is heated beyond a certain temperature T_c ("Curie Temperature") loses its magnetization, and then regains it when cooled down.

Fig. 11.



Curie's 1895 thesis, page 90 (orange low external magnetic field curve mine)

Pierre Curie
Nobel Prize 1903
(for radioactivity)

Main paradigm of phase transitions

Heuristic picture of phase transitions

A physical system has one or more **local order parameter(s)** x (*e.g.* $\vec{m}$). Assume:

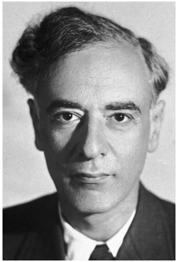
- The energy E is invariant under a symmetry group G

$$\forall g \in G, \quad E(g \cdot x) = E(x)$$

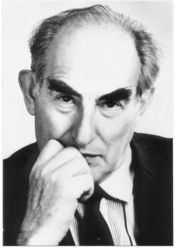
At high T the symmetry is preserved thanks fluctuations, at low T the symmetry is broken, one picks a single low E configuration out of the $|G|$ many available.

Examples

1. Crystallisation: $G = \mathbb{R}^3 \rtimes SO(3)$
2. Superconductivity: order $\psi \in \mathbb{C}$, $G = U(1)$



Lev Landau
Nobel Prize 1962



Vitaly Ginzburg
Nobel Prize 2003

The $d = 2$ splinter

Mermin-Wagner theorem (1966)

There is no possible spontaneous breaking of a continuous symmetry at finite T for systems with local interactions in $d = 2$

In other words: entropy always beats energy in $d = 2$ for a continuous symmetry (as in $d = 1$ for $\mathbb{Z}_2$)

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XY model

$$E = -\sum_{\langle i,j \rangle} \cos(\phi_i - \phi_j)$$

ϕ_i is an angle and $G = U(1)$

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Kosterlitz-Thouless phase transition **without** local symmetry breaking – **topological phase transition**

John Kosterlitz
Nobel Prize 2016

David Thouless
Nobel Prize 2003

Summary

1. Usually symmetries in causes $\implies$ symmetries in effects

2. But symmetries can be unstable (*e.g.* buckling of rails)
3. Adding thermal fluctuations can unbreak a symmetry [**entropy** vs **energy**]
4. Going from symmetry broken to unbroken as T increases is how most **phase transitions** work

5. There is more, with topological phase transitions