Relativistic continuous matrix product states

new results and perspectives

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RCMPS: *A variational ansatz to solve* $1+1d$ *relativitic QFT without discretization or cutoff and to arbitrary precision*

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Two papers

- Variational method in relativisitic QFT without cutoff (short)
  `arXiv:2102.07733v1`
- Relativistic continuous matrix product states for QFT without cutoff (long)
  `arXiv:2102.07741v1`

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RCMPS: *A variational ansatz to solve $1+1d$ relativitic QFT without discretization or cutoff and to arbitrary precision*

Two papers

- Variational method in relativisitic QFT without cutoff (short) `arXiv:2102.07733v1`
- Relativistic continuous matrix product states for QFT without cutoff (long) `arXiv:2102.07741v1`

New unpublished results soon to be in `v2`

- Computation of vertex operators
- Cost of optimization $\propto D^3 \implies$ numerically efficient

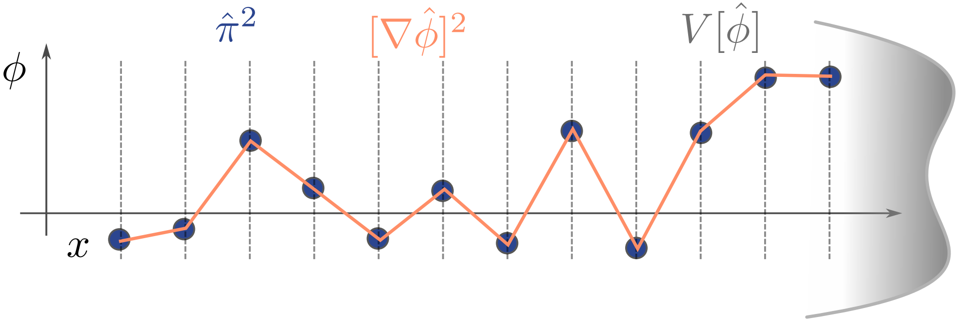
Outline

1. Scalar fields in $1+1$ dimensions
2. Solving by discretizing
3. Variational method in the continuum
4. Continuous matrix product states and their limitations
5. Relativistic twist $\psi \to a$
6. Making the numerics powerful $D^6 \to D^3$
7. Open questions

Basics of relativistic scalar field theory

from a condensed matter viewpoint

Intuitive definition: canonical quantization



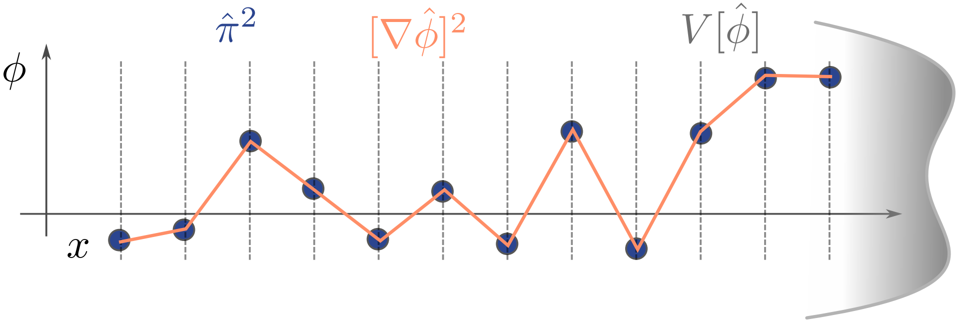
Hamiltonian

A continuum of nearest neighbor coupled anharmonic oscillators

$$\hat{H} = \int_{\mathbb{R}^d} \mathrm{d}^d x \quad \underbrace{\frac{\hat{\pi}(x)^2}{2}}_{\text{on-site inertia}} + \underbrace{\frac{[\nabla\hat{\phi}(x)]^2}{2}}_{\text{spatial stiffness}} + \underbrace{V(\hat{\phi}(x))}_{\text{on-site potential}}$$

with canonical commutation relations $[\hat{\phi}(x), \hat{\pi}(y)] = i\delta^d(x-y)\mathbb{1}$ (*i.e.* bosons)

Intuitive definition



Hilbert space

Fock space $\mathscr{H}_{\text{QFT}} = \mathscr{F}[L^2(\mathbb{R}^d)]$ – just like $x, p \to (a, a^\dagger)$ do $\hat{\pi}, \hat{\phi} \to \hat{\psi}, \hat{\psi}^\dagger$

$$|\Psi\rangle = \sum_{n=0}^{+\infty} \int \mathrm{d}x_1 \mathrm{d}x_2 \cdots \mathrm{d}x_n \underbrace{\varphi_n(x_1, x_2, \cdots, x_n)}_{\text{wave function}} \underbrace{\hat{\psi}^\dagger(x_1)\hat{\psi}^\dagger(x_2)\cdots\hat{\psi}^\dagger(x_n)}_{\text{local oscillator creation}} |\text{vac}\rangle$$

What are the problems compared to non-relativistic field theories

The Hamiltonian is ill defined on all states in the Hilbert space because of infinite zero point energy *i.e.* terms $\propto \hat{\psi}(x)\hat{\psi}^\dagger(x)$

$$\langle \Psi_1 | \hat{H} | \Psi_2 \rangle = \pm\infty \quad \text{and even} \quad \langle \text{vac} | \hat{H} | \text{vac} \rangle \propto \delta^d(0) = +\infty$$

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If the divergent vacuum terms are removed, the Hamiltonian is not bounded from below

$$\forall\ |\Psi\rangle \in \mathscr{H},\ \langle\Psi|\hat{H}_{\text{finite}}|\Psi\rangle = \text{finite but} \quad \exists\ \Psi_n \text{ s.t.} \lim_{n\to+\infty} \langle\Psi_n|H_{\text{finite}}|\Psi_n\rangle = -\infty$$

How are they are solved in the free case - Hamiltonian

Bogoliubov transform

Go from $\hat{\psi}(x), \hat{\psi}^\dagger(x)$ to $a(p), a^\dagger(p)$ with

$$a(p) = \frac{1}{\sqrt{2}}\left(\sqrt{\omega_p}\,\hat{\phi}(p) + i\frac{\hat{\pi}(p)}{\sqrt{\omega_p}}\right) \quad \text{with} \quad \omega_p = \sqrt{p^2 + m^2}$$

which yields

$$H_0 = \int \mathrm{d}p\,\omega_p\,\frac{1}{2}\left(a_p^\dagger a_p + a_p a_p^\dagger\right)$$

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Solution

- Take $H_{\text{QFT}} \equiv :H:_a$
- $|\text{free ground state}\rangle = |\text{vacuum}\rangle_a$
- $\mathscr{H}$ built from $a_{p_1}^\dagger \cdots a_{p_n}^\dagger |\text{vacuum}\rangle_a$

This solves the problematic free part exactly, and allows to define a finite interaction (in $1+1$)

Example: rigorous operator definition of ϕ_2^4

Renormalized ϕ_2^4 theory

$$H = \int \mathrm{d}x \, \frac{:\pi^2:_a}{2} + \frac{:(\nabla\phi)^2:_a}{2} + \frac{m^2}{2} :\phi^2:_a + g :\phi^4:_a$$

(note that $:\diamondsuit:_a$ depends on m)

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1. Rigorously defined relativistic QFT without cutoff (Wightman QFT)
2. Vacuum energy density finite
3. Very difficult to solve unless $g \ll m^2$ (perturbation theory)
4. Phase transition around $f_c = \frac{g}{4m^2} = 11$ i.e. $g \simeq 2.7$ in mass units

Hilbert spaces of RQFT in $1+1$

Two operator basis

The $\psi^\dagger(x)$ basis

Local oscillator basis

- $+$ Local in ϕ, π
- $+$ Natural for discretization
- $-$ Divergent and ill-defined

The $a_k^\dagger$ basis

"Relativistic" oscillator basis

- $-$ Non-local
- $-$ Less natural for discretization
- $+$ Regular and well-defined

Solving by discretizing

the state of the art

Example: Lattice ϕ_2^4

Defined by action:

$$S(\phi) = \sum_{\langle i,j \rangle} \underbrace{\frac{(\phi_i - \phi_j)^2}{2a^2}}_{(\nabla\phi)^2/2} a^2 + \sum_i \frac{1}{2}\mu_a^2 \phi_i^2 + \frac{1}{4}\lambda_a \phi_i^4$$

Taking the limit

The right way to get the continuum limit is to take:

$$\mu_a = \mu a^2 + \frac{3}{2}\log(a) a^2 \lambda$$
$$\lambda_a = \lambda a^2$$

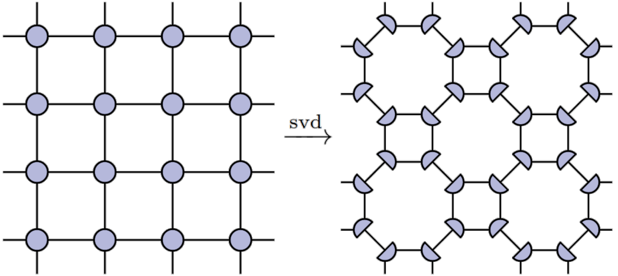
$\equiv$ normal ordering the interaction term $\equiv$ tadpole cancellation.

At 1st order in perturbation theory, $\phi^4 \propto \log(a^{-1})\phi^2$

Example with tensor network renormalization

Done with Clément Delcamp in 2020

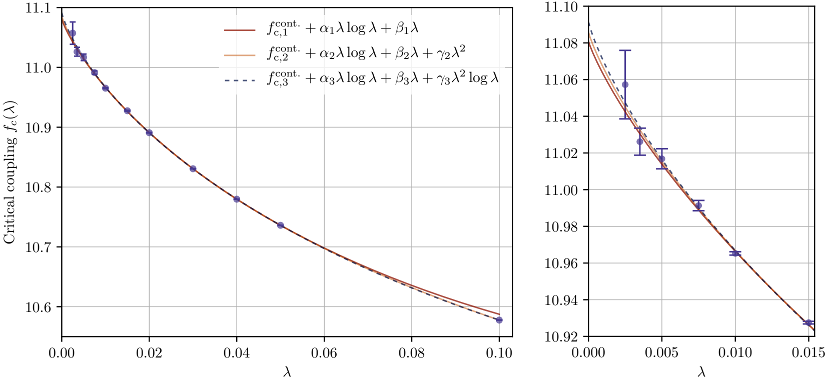
Discretize ϕ, write $Z = \sum S(\phi)$ as a tensor network and contract it with TRG + GILT



Technically: UV cutoff (lattice) and IR cutoff (number of RG steps)

Example with tensor network renormalization

Continuum limit taken **numerically**



More costly as the UV cutoff gets small because:

1. Field becomes unbounded at short distance $\rightarrow$ large starting bond dimension
2. More RG steps (with max χ) to get to the same scale

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The “numerical” continuum limit is expensive for relativistic QFT. Problem of local basis choice?

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1. UV fixed point is a free CFT $\implies$ continuum of singular values
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$\implies$ even theory independent: would apply to QCD (asymptotic freedom)

Results

For ϕ^4_2, critical coupling $f_c = \lambda/\mu^2$

Method	$f_c^{\text{cont.}}$	Year	Ref.
Tensor network coarse-graining	10.913(56)	2019	[9]
Borel resummation	11.23(14)	2018	[6]
Renormalized Hamil. Trunc.	11.04(12)	2017	[5]
Matrix Product States	11.064(20)	2013	[7]
Monte Carlo	11.055(20)	2019	[15]
This work	11.0861(90)	2020	

TABLE I. Comparison of several estimates of the critical coupling constant $f_c^{\text{cont.}}$ in the continuum obtained using different methods.

New results fresh from Ghent with MPS + finite entanglement scaling + continuum limit scaling $f_c = 11.09698(31)$ [arXiv:2104.10564]

see tilloy.wordpress.com for a discussion

The variational method

in the continuum

The variational method

In the Hamiltonian formulation:

- Guess a **finite dimensional submanifold** $\mathscr{M}$ of the QFT Hilbert space $\mathscr{H}$
- Find the ground state by minimizing $\langle H \rangle$:

$$|\text{ground}\rangle \simeq |\psi\rangle = \underset{\mathscr{M}}{\text{argmin}} \frac{\langle \psi | H | \psi \rangle}{\langle \psi | \psi \rangle}$$

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Example: naive Hamiltonian truncation

With an IR cutoff, momenta are discrete. Take as submanifold $\mathscr{M}$ the **vector space** spanned by:

$$a^\dagger_{k_1} a^\dagger_{k_2} \cdots a^\dagger_{k_r} |0\rangle_a$$

where $r \leqslant r_{\text{max}}$ and $k \leqslant k_{\text{max}}$ (one possible truncation)

Feynman's objection

Feynman's requirement for variational wavefunctions in RQFT (1987)

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All methods so far break one at least:

- Hamiltonian truncation fails at 1 (but works fairly well through its renormalized refinements)
- Tensor networks succeed at 1 and 2 but fail (a priori) at 3

Continuous matrix product states

Continuous Matrix Product States

Introduced by Verstraete and Cirac in 2010

Definition

$$|Q, R\rangle = \mathrm{tr}\left[\mathcal{P}\exp\left\{\int_0^L dx\; Q\otimes\mathbb{1} + R\otimes\psi^\dagger(x)\right\}\right]|0\rangle_\psi$$

- Q, R are $D\times D$ matrices,
- The trace is taken over this matrix space
- $[\psi(x), \psi^\dagger(y)] = \delta(x-y)$
- $\psi^\dagger(x)$ is non-relativistic creation operator (i.e. $\phi(x) = \frac{1}{\sqrt{2\nu}}[\psi(x)+\psi^\dagger(x)]$)
- $|0\rangle_\psi$ is the associated Fock vacuum

Idea:

- From MPS: a continuum limit
- From QFT: a sort of generalized “non-commutative” coherent state

Computations

Some correlation functions

$$\left\langle \hat{\psi}(x)^{\dagger}\hat{\psi}(x) \right\rangle = \mathrm{Tr}\left[e^{TL}(R \otimes \overline{R})\right]$$

$$\left\langle \hat{\psi}(x)^{\dagger}\hat{\psi}(0)^{\dagger}\hat{\psi}(0)\hat{\psi}(x) \right\rangle = \mathrm{Tr}\left[e^{T(L-x)}(R \otimes \overline{R})e^{Tx}(R \otimes \overline{R})\right]$$

$$\left\langle \hat{\psi}(x)^{\dagger}\left[-\frac{d^2}{dx^2}\right]\hat{\psi}(x) \right\rangle = \mathrm{Tr}\left[e^{TL}([Q, R] \otimes [\overline{Q}, \overline{R}])\right]$$

with $T = Q \otimes \mathbb{1} + \mathbb{1} \otimes \bar{Q} + R \otimes \bar{R}$

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Example

Lieb-Liniger Hamiltonian

$$\mathcal{H} = \int_{-\infty}^{+\infty} dx \left[\frac{d\hat{\psi}^\dagger}{dx}\frac{d\hat{\psi}}{dx} - \mu \hat{\psi}^\dagger \hat{\psi} + c \hat{\psi}^\dagger \hat{\psi}^\dagger \hat{\psi} \hat{\psi}\right]$$

Solve by **minimizing**: $\langle Q, R | \mathcal{H} | Q, R \rangle = f(Q, R)$

State of the art on CMPS

Contrary to common beliefs, CMPS are fairly efficient

1. Fully variational calculations at $D = 256$ by Ganahl-Rincon-Vidal 2016
2. Recently Tuybens-De Nardis-Haegeman-Verstraete `arXiv:2006.01801` included open-boundaries efficiently

Standard CMPS and relativistic fields

Applying cMPS to e.g. the ϕ^4 Hamiltonian

$$\langle Q, R | \widehat{h}_{\phi^4} | Q, R \rangle = \infty$$

Oh no!

The short distance behavior of cMPS is the wrong one, even the free theory is hard to approximate.

A possible fix by Haegeman-Cirac-Osborne-Verschelde-Verstraete 2010:

$$H \to H_\Lambda := H + \frac{1}{\Lambda^2} \int \mathrm{d}x \, \frac{(\partial_x \pi)^2}{2}$$

Going relativistic

Changing of operator basis

Towards relativistic CMPS

Local basis in position of the QFT: $\psi^\dagger$, ϕ, π, $|0\rangle_\psi$
Diagonal basis of the free part: $a_k^\dagger$, $|0\rangle_a$

Bogoliubov transform

Go from $\hat{\psi}(x), \hat{\psi}^\dagger(x)$ to $a(p), a^\dagger(p)$ with

$$a(p) = \frac{1}{\sqrt{2}}\left(\sqrt{\omega_p}\,\hat{\phi}(p) + i\frac{\hat{\pi}(p)}{\sqrt{\omega_p}}\right) \quad \text{with} \quad \omega_p = \sqrt{p^2 + m^2}$$

which yields

$$H_0 = \int \mathrm{d}p\, \omega_p\, \frac{1}{2}\left(a_p^\dagger a_p + a_p a_p^\dagger\right)$$

Go from $|0\rangle_\psi$ to $|0\rangle_a$
and
Go from $\psi(x)$ to $a(x) = \int \mathrm{d}p\; a(p)\mathrm{e}^{ipx} \neq \psi(x)$

Relativistic CMPS

Definition

$$|R, Q\rangle = \mathrm{tr}\left\{\mathcal{P}\exp\left[\int \mathrm{d}x\, Q\otimes \mathbb{1} + R\otimes a^\dagger(x)\right]\right\}|0\rangle_a$$

Some properties

1. $|0,0\rangle = |0\rangle_a$ is the ground state of H_0 hence exact CFT UV fixed point (because interaction super-renormalizable)
2. $\langle Q, R|h_{\phi^4}|Q, R\rangle$ is finite for all Q, R (not trivial)

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$a(x)$ is not covariant but the state cannot be exactly Poincaré invariant anyway!

Consequence on the Hamiltonian

Hamiltonian density in $a(x)$ basis

H is local in $\psi(x)$, not in $a(x)$...

$$
\begin{aligned}
H = &\int dx_1 dx_2 D(x_1 - x_2) a^\dagger(x_1) a(x_2) \\
&+ \int dx_1 dx_2 dx_3 dx_4 K(x_1, x_2, x_3, x_4) a(x_1) a(x_2) a(x_3) a(x_4) + 4a^\dagger aaa + 3a^\dagger a^\dagger aa \\
&+ \text{h.c.}
\end{aligned}
$$

But fortunately exponentially decreasing: K decays $\propto e^{-m|x|}$ for $|x| \gg m$.

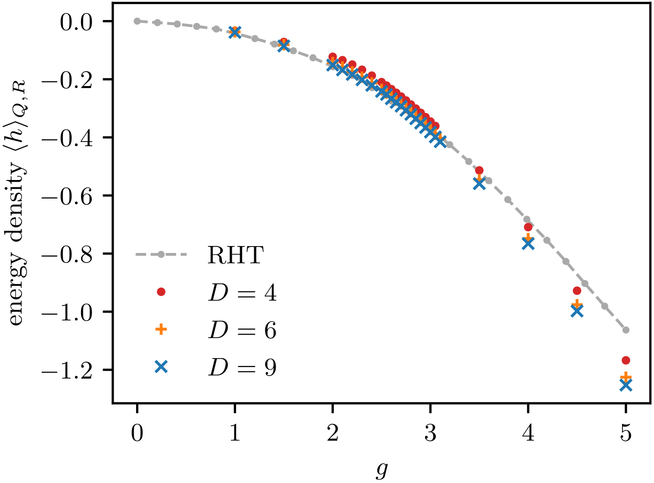
The variational algorithm

Procedure:
Compute $e_0 = \langle Q, R | h_{\phi^4} | Q, R \rangle$ and $\nabla_{Q,R} e_0$
Minimize e_0 with **TDVP** aka gradient descent with a metric

Computations of e_0 and ∇e_0in a nutshell:

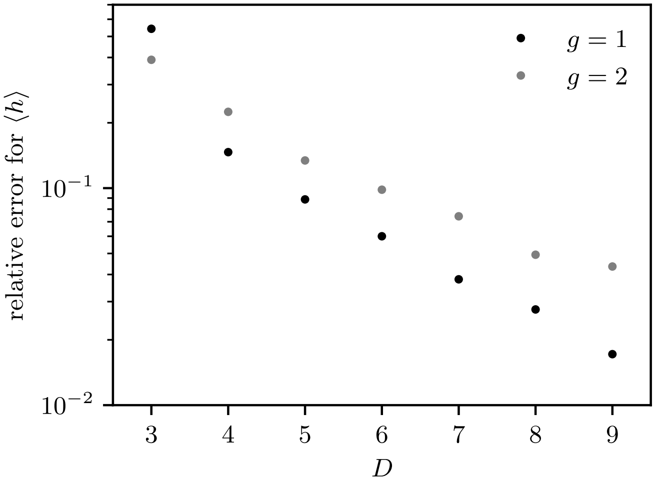
1. Contains an algebraic part identical to standard cMPS
2. Involves quadruple integrals without analytic solutions
   Initial v1 idea: compute the integrals with Quadpack

Initial results



Compared with the Renormalized Hamiltonian Truncation results of Rychkov and Vitale from 2015.

Results



Compared with the “high precision” Renormalized Hamiltonian Truncation results of Elias Miro, Rychkov, and Vitale from 2017 for $g = 1$ and $g = 2$.

Scaling comparison with renormalized Hamiltonian truncation

Ren. Hamiltonian truncation

IR cutoff L, energy truncation E_T

- Uses a vector space
- Function to minimize is quadratic, hence linear problem
- Number of parameters $\propto \mathrm{e}^{L \times E_T}$
- Error $\propto 1/E_T^3$

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Relativistic CMPS

entanglement truncation D

- Uses a manifold
- Minimization is a priori non-trivial but doable
- Number of parameters $\propto D^2$
- Error $o(1/D^\alpha)$, $\forall\ \alpha$ (folklore)

Improving the algorithm

Computing vertex operators

Main insight

$\langle : e^{b\phi(x)} : \rangle_{QR}$ computable by solving an ODE with cost $\propto D^3$

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$\langle :e^{b\phi(x)}: \rangle_{QR}$ computable by solving an ODE with cost $\propto D^3$

Going from $\phi(x)$ to $a(x)$ gives:

$$\begin{aligned}\langle :e^{b\phi(0)}: \rangle_{QR} &= \left\langle \exp\left[b\int J(x)a^\dagger(x)\right]\exp\left[b\int J(x)a(x)\right]\right\rangle_{Q,R} \\ &= Z_{bJ,bJ}\end{aligned} \tag{1}$$

with

$$J(x) = \int \mathrm{d}k \frac{1}{\sqrt{2\omega_k}} e^{ikx} \tag{2}$$

and Z_{j_1,j_2} is just the generating functional

$$Z_{j_1,j_2} = \mathrm{tr}\left[\mathcal{P}\exp\int \mathbb{T} + j_1(x)R\otimes\mathbb{1} + +j_2(x)\mathbb{1}\otimes\bar{R}\,\mathrm{d}x\right] \tag{3}$$

Algorithm v2 $\propto D^3$

1. Compute $Z_{bJ,bJ}$ by solving the ODE

$$\partial_x \rho = \mathcal{L}\rho + bJ(x)(R\rho + \rho R^\dagger)$$

and taking the trace at $x = +\infty$

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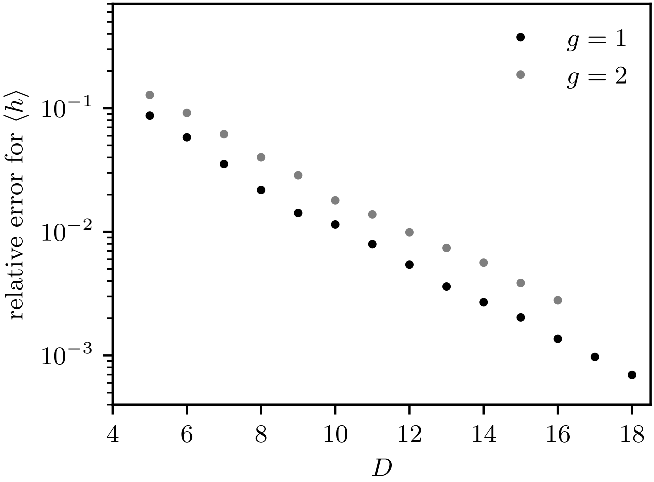
Bottom line

Solve with cost $\propto D^3$ all theories with $V(\phi)$ poly $: \phi^n :$ or exponential $: e^{b\phi} :$ (including Sine/Sinh-Gordon and thus Fermionic theories via bosonization)

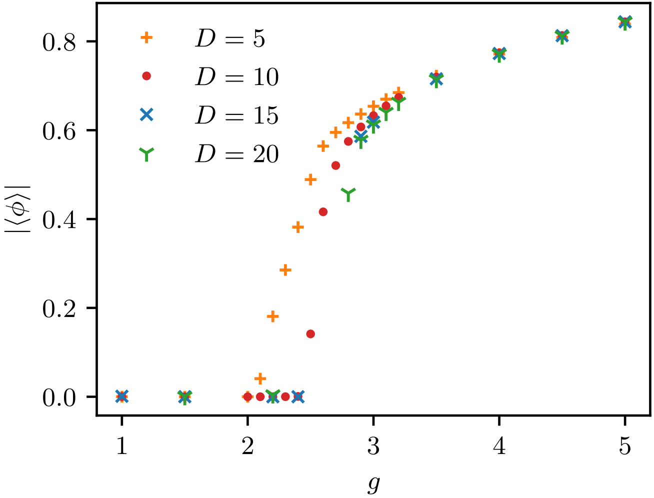
New results



Approximately exact value extrapolated from $D = 25$ (boostrapped error $< 10^{-4}$). More precise than high precision RHT. Pushable to $D > 40$

New results

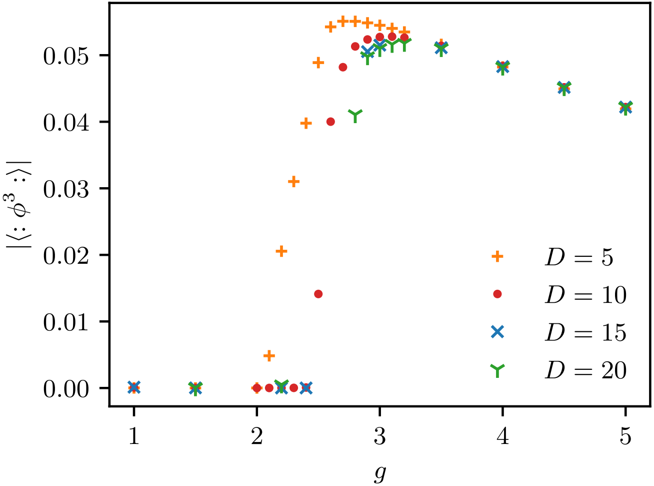
Magnetization $\langle\phi\rangle$



Some points near criticality missing because computations not yet finished...

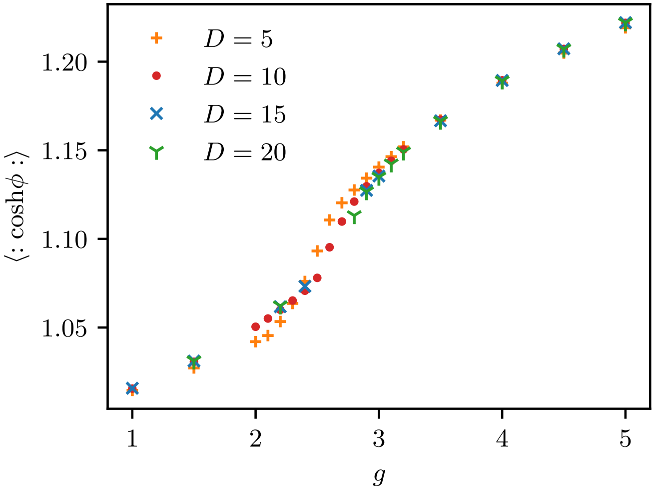
New results

$\langle : \phi^3 : \rangle$



New results

$\langle : \cosh(\phi) : \rangle$



Open problems and perspectives

New entanglement entropy

Conjecture

For the notion of space locality is induced by $a^\dagger(x), a(x)$ (instead of usual $\phi(x)$), gapped QFT ground states verify the area law with a **finite** prefactor.

- This entanglement entropy is weird from a relativistic point of view
- But captures the notion of approximability with tensor network states

Useful notion? Can the conjecture be proved?

More general short distance behavior

RCMPS have the short distance behavior of a free CFT (fairly generic in HEP)

Can one deal with relevant perturbations of other UV CFTs (e.g. Ising)?

Equivalent of $a(x)$? Coulomb gas construction?

Relativistic CMERA

MERA is non-relativistic (not a CFT) at short distance

Is RCMERA possible? I.e. CMERA for *critical* RQFT

$$\langle\psi_{\text{rcmera}}|\phi(x)\phi(y)|\psi_{\text{rcmera}}\rangle \underset{|x-y|\to 0}{\sim} \frac{1}{|x-y|^{2\Delta_1}} \quad \text{[UV CFT]}$$

$$\langle\psi_{\text{rcmera}}|\tilde{\phi}(x)\tilde{\phi}(y)|\psi_{\text{rcmera}}\rangle \underset{|x-y|\to +\infty}{\sim} \frac{1}{|x-y|^{2\Delta_2}} \quad \text{[IR CFT]}$$

Higher dimensions

RQFT difficulty

Normal ordering / tadpole cancellation no longer sufficient
Whightman QFT still have Hilbert space, but less explicit (not free Fock space)

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(non-relativistic) Tensor network difficulty

Continuous tensor network states less developed in $2+1$

1. Proposal with Ignacio Cirac: R, Q promoted to fields, needed to preserve Euclidean invariance
2. Successfully tested on Gaussian problems with Teresa Karanikolaou (also independently in Ghent by Bastiaan Aelbrecht)
3. Need to solve a boundary $1+1$ RQFT to compute more general expectation values

Non-relativistic $2+1$ now seems feasible thanks to RCMPS...

Summary

1. Ansatz for $1+1$ relativistic QFT
2. No cutoff, UV or IR
3. UV is captured exactly even at $D=0$
4. Efficient (cost poly D, error at most superpoly $1/D$) and now competitive
5. Rigorous (variational)