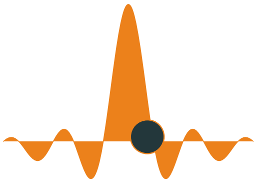
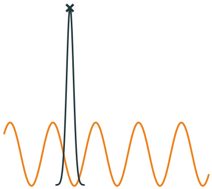
Non-Markovian wave-function collapse models are Bohmian-like theories in disguise

Antoine Tilloy, with Howard M. Wiseman

arXiv:2105.06115





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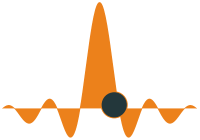
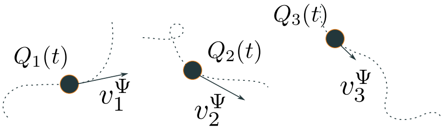
Introduction: two realist reconstructions of QM

Bohm



- *“Particles move”*



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$$\partial_t |\psi\rangle = -iH|\psi\rangle$$

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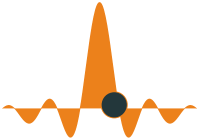

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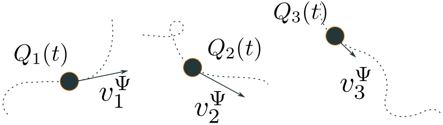

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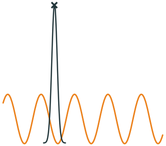
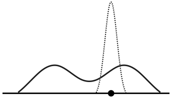
Collapse



- The wavefunction does **not** evolve unitarily

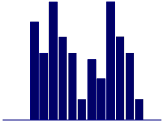
$$\partial_t|\psi\rangle = -iH|\psi\rangle + \varepsilon\, f(\psi, w)\,|\psi\rangle$$

- Beables are constructed from w or ψ



Introduction: differences

1. **Empirically different**



$\rightarrow$ collapse model predictions deviate from those of the **Standard Model**

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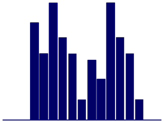

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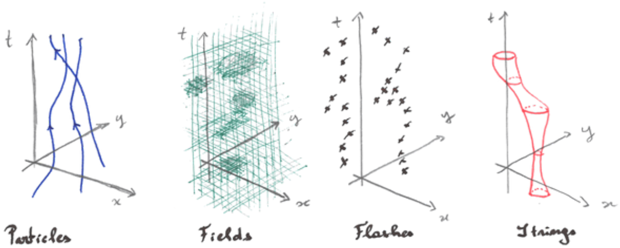
2. **Ontologically different**



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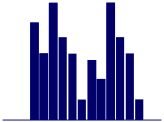

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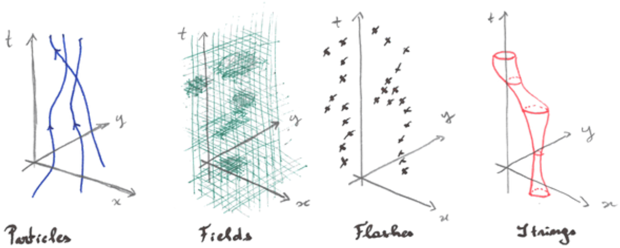
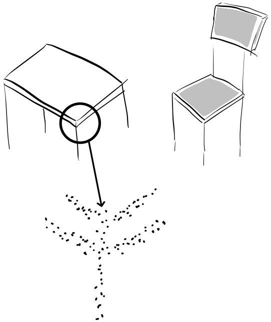


3. **Different spirit**
   “Modify the Schrödinger equation” *vs* “Add hidden variables”

Introduction: similarities

1. Theories with a **primitive ontology** – [Allori et al. 2008, 2013]



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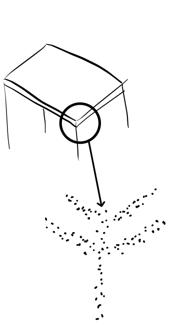

2. Dynamical reduction works in similar ways – [Toros et al. 2016]

$$\psi_C^{\text{Bohm}}(x_{\text{bath}}, \cdot) \sim \psi^{\text{GRW}}(\cdot)$$

Weak equivalence

Empirically: collapse $=$ Bohm on system $+$ bath

For a given collapse model on a system S, there exists a bath B such that Bohmian mechanics on $\{S + B\}$ has the same empirical content.

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For a given collapse model on a system S, there exists a bath B such that Bohmian mechanics on $\{S + B\}$ has the same empirical content.

1. Collapse model dynamics has to be linear at the master equation level
   $\rho_t = \mathbb{E}\Big[\, |\psi_t\rangle\langle\psi_t| \,\Big]$

$$\rho_t = \Phi_t \cdot \rho_0$$

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$$\exists\ H \ \text{ and } \ |\Psi_0\rangle = |\psi_0\rangle \otimes |\text{aux}\rangle \ \text{ s.t. } \ \partial_t |\Psi_t\rangle = -iH|\Psi_t\rangle \ \text{ and } \ \text{tr}_B\Big[\,|\Psi_t\rangle\langle\Psi_t|\,\Big] = \rho_t$$

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4. This dilated unitary dynamics can be Bohmianized

Strong equivalence

Metaphysically: collapse = Bohm on system + bath

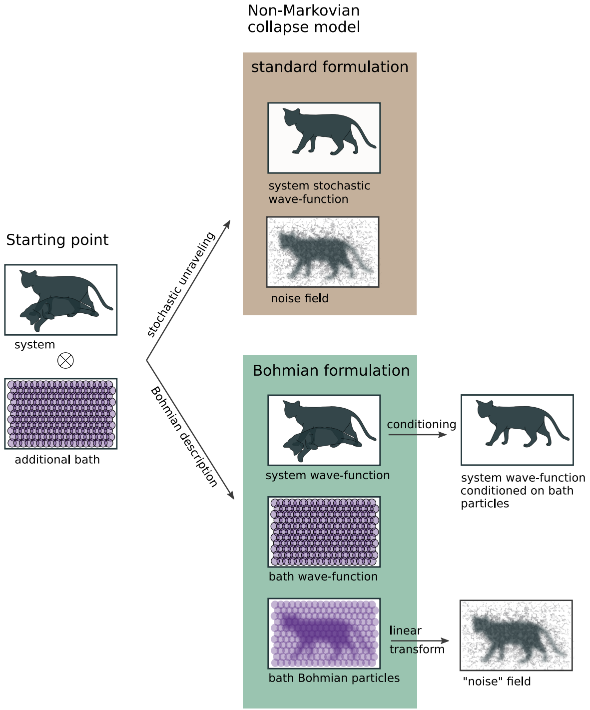
For a given collapse model on a system S, there exists a bath B such that Bohmian mechanics on $\{S+B\}$ **contains the exact same objects**

- The Bohmian wave-function on $S+B$ conditioned on bath hidden variables **is** the collapse model stochastic wave function:

$$|\psi^{\text{collapse}}\rangle = |\psi_C^{\text{Bohm}}(x_{\text{bath}})\rangle$$

- The Bohmian hidden variables of the bath are in one to one correspondence with the collapse noise

$$x_{\text{bath}}^{\text{Bohm}} \underset{\text{linear transform}}{\longleftrightarrow} \textbf{collapse noise} \;\; w$$


Non-Markovian
collapse model
standard formulation
system stochastic
wave-function
noise field
Starting point
system
additional bath
stochastic unraveling
Bohmian description
Bohmian formulation
system wave-function
conditioning
system wave-function
conditioned on bath
particles
bath wave-function
bath Bohmian particles
linear
transform
"noise" field

Precise formulation

Collapse model

For states $|\psi\rangle$ in a Hilbert space $\mathscr{H}_s$, we are given a collapse model "collapsing" a set of operators A_i, $i = 1 \cdots d$, with a colored noise field $w_i(t)$:

$$\partial_t |\psi_w\rangle = \underbrace{-iH|\psi\rangle}_{\text{linear QM}} + \underbrace{f\left(A_i, w_i, \psi_w\right)|\psi_w\rangle}_{\text{collapse term}}$$

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Bohmian theory on extended space

One can find a bath Hilbert space $\mathscr{H}_b$ made of interacting harmonic oscillators $\{x_{i,\omega}\}$ indexed by $i = 1 \cdots d$ and a frequency $\omega \in \mathbb{R}$, and a (reasonable) interaction Hamitonian H_{int} such that:

1. $|\Psi(0)\rangle = |\psi(0)\rangle \otimes |0\rangle \in \mathscr{H}_{\text{total}} = \mathscr{H}_s \otimes \mathscr{H}_b$
2. $|\psi^C(t)\rangle = \langle \{x_{i,\omega}(t)\} | \Psi_t\rangle \in \mathscr{H}_s$
3. For $x_{i,\omega}(0)$ drawn with the **Born rule**, $|\psi^C\rangle$ has the same law as $|\psi_w\rangle$

Comments

1. Technically, the proof uses an “old” result
   → [Gambetta & Wiseman 2003]
2. The result is **valid** for continuous collapse models with **non-white noise**
3. The result is still valid for continuous collapse models with white noise, but unimpressive – Bohmian description far more complicated
4. The result is obtained in a bruteforce way, with an explicit non-trivial computation

General continuous collapse models

One starts with a **non-linear** stochastic Schrödinger equation:

$$\partial_t|\psi\rangle = -iH|\psi\rangle + \underbrace{\varepsilon\, f(\psi, w)\, |\psi\rangle}_{\text{non-linear}} \tag{1}$$

Constraints on f are obtained by:

- Requiring that equation (1) reduces superpositions of certain operators (position related).
- Imposing a **linear** master equation for $\rho_t = \mathbb{E}\Big[\,|\psi_t\rangle\langle\psi_t|\,\Big]$:

$$\rho_t = \Phi_t \cdot \rho_0 \quad CPTP$$

Required for the probabilistic interpretation (Born rule, no faster-than-light signalling).

Collapse models with colored noise: linear evolution

Linear differential equation

$$\frac{\mathrm{d}}{\mathrm{d}t}|\phi_w(t)\rangle = \Big[-iH + \underbrace{\sqrt{\gamma}\, w_i(t) A_i}_{\text{noise drive}} \underbrace{- 2\sqrt{\gamma}\, A_i \int_0^t \mathrm{d}s\, D_{ij}(t,s) \frac{\delta}{\delta w_j(s)}}_{\text{weird necessary memory term}} \Big] |\phi_w(t)\rangle,$$

with **colored** noise field: $\mathbb{E}\left[w_i(t) w_j(s)\right] = D_{ij}(t,s)$

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- **Master equation**: For $\rho_t = \mathbb{E}\Big[\, |\phi_w(t)\rangle\langle\phi_w(t)| \,\Big]$, on has $\rho_t = \Phi_t \cdot \rho_0$ with Φ_t Completely Positive Trace Preserving (CPTP)
- **Dilation**: the master equation admits a well known unitary dilation with a **non-Markovian** bosonic bath

Collapse models with colored noise: non-linear

Normalization

$$|\psi_w(t)\rangle = \frac{1}{\sqrt{\langle\phi_w(t)|\phi_w(t)\rangle}}|\phi_w(t)\rangle$$

but now $|\psi_w(t)\rangle$ does not yield a linear master equation.

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Noise tilt

Restore the linear master equation by biasing the noise field

$$w_j^{[t]}(s) = w_j(s) + 2\sqrt{\gamma}\int_0^t \mathrm{d}\tau\, D_{ij}(\tau, s)\langle A_i\rangle_\tau,$$

Full collapse model $=$ linear equation $+$ normalization $+$ noise drift

Intuition for the proof of the equivalence

Linear collapse equation = conditioning on **fixed** Bohmian hidden variables

$$\frac{\mathrm{d}}{\mathrm{d}t}|\phi_w(t)\rangle \underset{\text{collapse}}{:=} \left[-iH + \sqrt{\gamma}\, w_i(t) A_i - 2\sqrt{\gamma}\, A_i \int_0^t \mathrm{d}s\, D_{ij}(t,s) \frac{\delta}{\delta w_j(s)}\right] |\phi_w(t)\rangle$$

$$|\phi_w(t)\rangle \underset{\text{Bohm}}{:=} \langle w|\Psi\rangle_{S+B} \quad \underset{\text{conditioning}}{\longleftarrow} \quad |\Psi\rangle_{S+B} \quad \text{complete wavefunction}$$

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Non-trivial continuous noise drift = Bohmian guiding law

$$w_j^{[t]}(s) \underset{\text{collapse}}{:=} w_j(s) + 2\sqrt{\gamma} \int_0^t \mathrm{d}\tau\, D_{ij}(\tau, s) \langle A_i \rangle_\tau$$

$$\frac{\mathrm{d}}{\mathrm{d}t} w_j^{[t]}(s) \underset{\text{Bohm}}{:=} \mathcal{V}(\psi, w)$$

Lessons & questions

1. We should be careful of distinguishing **quantum theory** from its instantiation in the **Standard Model**
2. The two leading approaches to *down to earth* foundations of quantum theory are formally the same: **not** a duality, **not** even just empirically equivalent, ontologies in one to one
3. The distinction between a **deterministic** and a **stochastic** theory is inevitably blurry
   - In collapse models randomness is progressively revealed
   - In Bohmian mechanics it is in the initial condition
4. Is there another way to construct realist interpretations without **guiding laws**?