Exploring the Metaphysical and Practical Implications of Quantum Theory

DALL·E "Two cats falling into a blackhole in synthwave style, digital art"

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April 4th 2023

EconophysX seminar

MINES PARIS | PSL

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Inria

Famous people say absolutely insane things...

"We know that the moon is **demonstrably** not there when nobody looks"

David Mermin - 1981

But quantum mechanics is powerful!

Strong physical Church Turing Thesis

Everything that can be **efficiently** computed by a physical machine can be **efficiently** computed by a Turing machine.

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Strong physical Church Turing Thesis

Everything that can be **efficiently** computed by a physical machine can be **efficiently** computed by a Turing machine.

Example: factoring

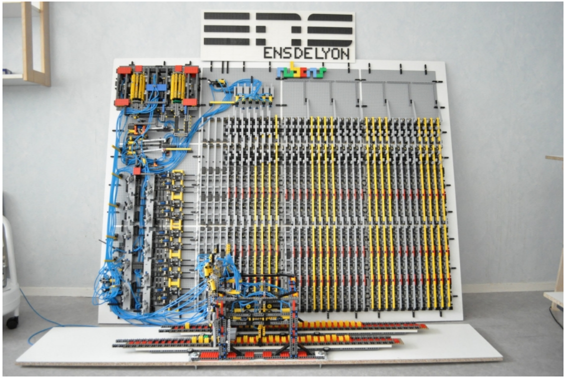
$$\underbrace{19209192\cdots 001}_{n \text{ digits}} = p \times q$$

Finding p and q can be done in time T

$$T \propto \exp\left(n^{1/3}\right)$$

with the General number field sieve (best algorithm known)

But quantum mechanics is powerful!



ENS Lyon – Lego Turing machine $\sim 10^{-2}$flops

Oak ridge – Summit $\sim 10^{17}$ flops

$$t_{\text{Lego}} = C_{\text{Lego}} \exp\left(n^{1/3}\right)$$
$$t_{\text{Summit}} = C_{\text{Summit}} \exp\left(n^{1/3}\right)$$

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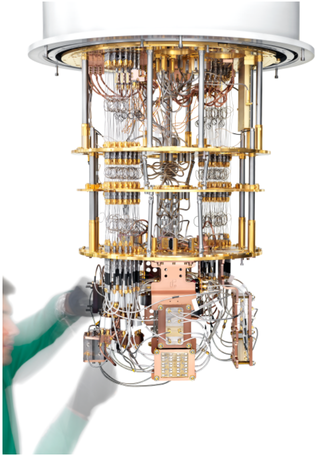

- Turing Machines with best algorithm

$$t = C \exp\left(n^{1/3}\right)$$

- Shor's algorithm on quantum bits

$$t \propto n^3$$

Quantum computers will break the strong form of the Church-Turing Thesis

Outline

- Postulates (the rules of the game)
- The measurement problem (the metaphysical problem)
- Opening the box (aka interpretations)
- Bell's inequality (the practical problem)

The postulates

DALL·E "A mysterious black box, on the floor, in the middle of an empty room"

Postulate 1: the world and its dynamics

Kinematics

The state of a system $|\psi\rangle$ is a norm 1 vector in a **separable Hilbert space** $\mathscr{H}$

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Dynamics – Schrödinger equation

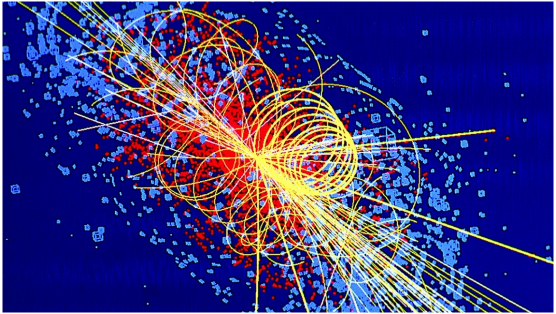
The state vector $|\psi\rangle$ evolves unitarily

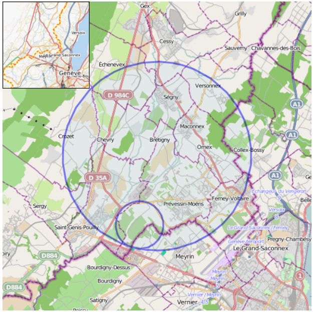
$$\frac{\mathrm{d}}{\mathrm{d}t}|\psi_t\rangle = -\frac{i}{\hbar}H|\psi_t\rangle$$

where H is the **Hamiltonian**, a self-adjoint operator on $\mathscr{H}$

Then, an effective model of a system, or fundamental model of the world, is given by a choice of $\mathscr{H}$ and H

The standard model of particle physics





The **standard model** is an instantiation of quantum theory that is the most fundamental we know (without gravity)

1. Hilbert space $\mathscr{H}$ (the fundamental particles and their statistics)
2. Hamiltonian H (all the forces/interactions between the particles)

Comment: in practice $\mathscr{H}$ is a large tensor product

Tensor product structure

In the standard model and most effective descriptions, $\mathscr{H}_{\text{tot}}$ splits into small Hilbert space for each subsystem Σ_i

$$\mathscr{H} = \bigotimes_i \mathscr{H}_{\Sigma_i}$$

The joint state of two atoms is in $\mathscr{H}_{\text{atom 1}} \otimes \mathscr{H}_{\text{atom 2}}$

$$e.g. \quad |\psi_{12}\rangle = |\text{ground}\rangle_1 \otimes |\text{excited}\rangle_2$$

But a normalized superpositions can also be generated from the dynamics H

$$|\psi_{\text{EPR}}\rangle = \frac{1}{\sqrt{2}}\left(|\text{ground}\rangle_1 \otimes |\text{excited}\rangle_2 - |\text{excited}\rangle_1 \otimes |\text{ground}\rangle_2\right)$$

This state is **entangled**.

Postulate 2: measurement

Measurement postulate

For a system "described" by $|\psi\rangle \in \mathscr{H}$ and a measurement "described" by projector Π_i such that $\sum_i \Pi_i = \mathbb{1}$ one has:

Born rule :

Result " i " with probability $\mathbb{P}[i] = \langle\psi|\Pi_i|\psi\rangle$

Collapse :

$$|\psi\rangle \longrightarrow \frac{\Pi_i|\psi\rangle}{\sqrt{\mathbb{P}[i]}}$$

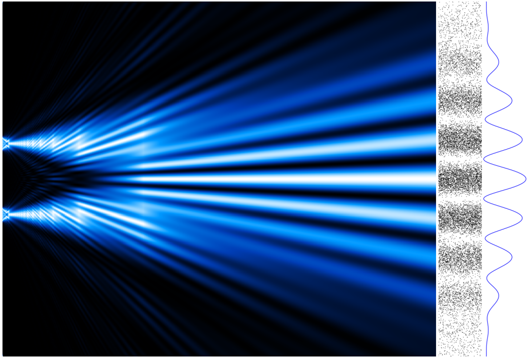
Max Born 1926

John von Neumann 1932

Example: interference

For a single non-relativistic particle (*e.g.* electron, neutron)

1. $\mathscr{H} = L^2(\mathbb{R}^3, \mathbb{C})$
2. $H = -\frac{\hbar^2}{2m}\Delta$
3. Measure X, Y, Z on the screen



An operational framework

A priori, the postulates say **nothing** about the fabric of the world:

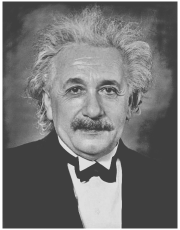
- $|\psi\rangle$ is a *a priori* only a tool
- Measurement results are the only thing we see!

The measurement problem

Making the black box a theory of the world?

The need for a measurement postulate is weird

- What counts as a measurement?
- How can measurement be primitive?
- What are measurements made of? (What is real?)
- Can postulate 2 (measurement) be derived from postulate 1 (dynamics)



Albert Einstein 1935

notions of 'reversible' and 'irreversible'. Einstein said that it is theory which decides what is 'observable'. I think he was right – 'observation' is a complicated and theory-laden business. Then that notion should not appear in the *formulation* of fundamental theory. *Information? Whose* information? Information about *what*?

On this list of bad words from good books, the worst of all is 'measurement'. It must have a section to itself.

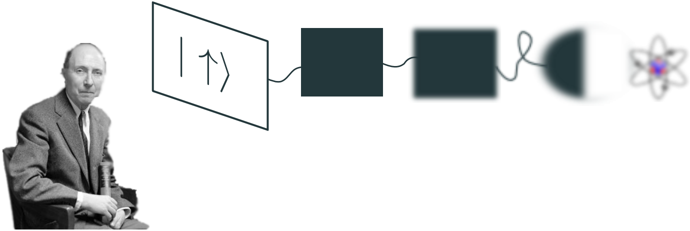
Physics World, Against Measurement

John S. Bell 1989

Is it measurement or dynamics?

Heisenberg cut

Split between the **system**, evolving with linear dynamics and the **observer** who can apply the non-linear measurement postulate



Eugene Wigner

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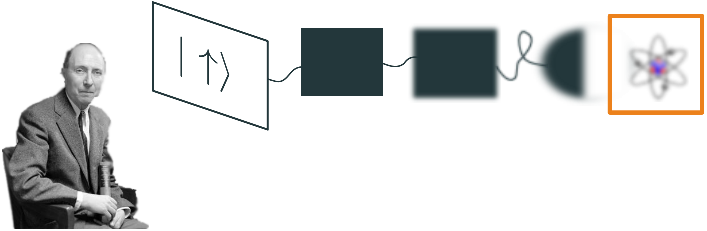

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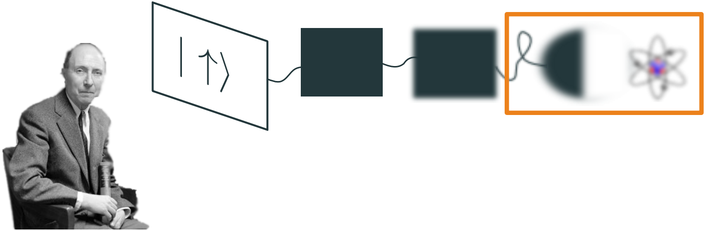

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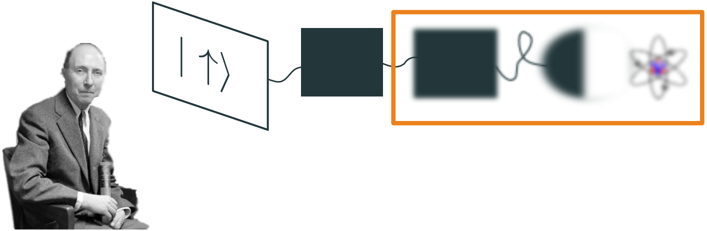

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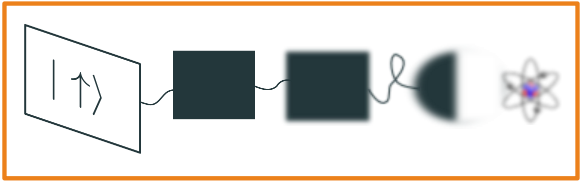

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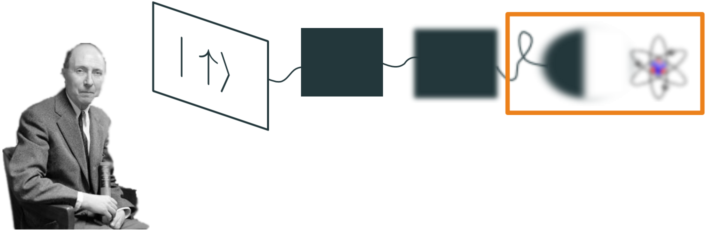

Eugene Wigner

Deriving 2 from 1?

Describe the quantum detector quantum mechanically adding $\mathscr{H}_{\text{detector}}$ and some H coupling detector and system.

We need

$$|\text{ground}\rangle \otimes |\text{detector}\rangle \underset{H}{\longrightarrow} |\text{whatever 1}\rangle \otimes |\text{detector not triggered}\rangle$$
$$|\text{excited}\rangle \otimes |\text{detector}\rangle \underset{H}{\longrightarrow} |\text{whatever 2}\rangle \otimes |\text{detector triggered}\rangle$$

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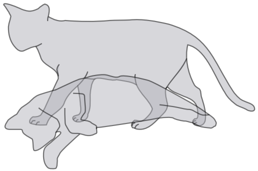
But this implies, by **linearity** of the dynamics

$$\Big(\alpha\,|\text{ground}\rangle + \beta\,|\text{excited}\rangle\Big) \otimes |\text{detector}\rangle$$
$$\xrightarrow[H]{} \alpha\,|\text{whatev 1}\rangle \otimes |\text{not triggered}\rangle + \beta\,|\text{whatev 2}\rangle \otimes |\text{triggered}\rangle$$

Superpositions propagate from micro to macro, never “collapsing”

Schrödinger's cat



$$\Big(\alpha\,|\text{ground}\rangle + \beta\,|\text{excited}\rangle\Big) \otimes |\text{detector}\rangle \otimes |\text{cat}\rangle \otimes |\text{me}\rangle$$

$$\xrightarrow[H]{} \quad \alpha\,|\text{whatev 1}\rangle \otimes |\text{not trigg}\rangle \otimes |\text{cat alive}\rangle \otimes |\text{``cool it's alive''}\rangle$$

$$+\ \beta\,|\text{whatev 2}\rangle \otimes |\text{triggered}\rangle \otimes |\text{cat dead}\rangle \otimes |\text{``oh no it's dead''}\rangle$$

Decoherence solves the problem for all practical purposes

Macroscopic systems like detectors, cats, or people have colossally large $\mathscr{H}_{\text{big}}$

$$|\text{ground}\rangle \otimes |\text{big system}\rangle \xrightarrow[H]{} |\text{whatever 1}\rangle \otimes |\text{big system 1}\rangle$$

$$|\text{excited}\rangle \otimes |\text{big system}\rangle \xrightarrow[H]{} |\text{whatever 2}\rangle \otimes |\text{big system 2}\rangle$$

For a generic H: $\langle \text{big system 1} \mid \text{big system 2}\rangle \simeq 0 \leftarrow$ **"decoherence"**

Decoherence

Decoherence explains why the dead cat and the live cat do not produce interferences with each other, but does not explain why only one **exists**.

A consistent blackbox: the orthodox interpretation

Decoherence shows that different Heisenberg cuts do not lead to contradictions as long as as they are placed at a sufficiently macroscopic level.

$$\Longrightarrow$$

The black box is self-consistent but **the measurement problem remains**.

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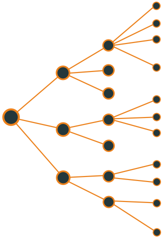
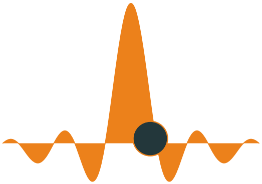
Reactions

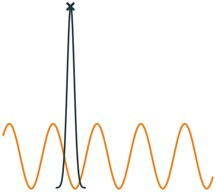
1. Pragmatic orthodoxy: *Shut up and calculate!*
2. Sectarian orthodoxy: Physics should only talk about what is *observable*, the rest *does not exist*
3. Fashionable orthodoxy: there *is* only information about the world

3 possible ways to see through the blackbox

1. Accept that the branches of $|\psi\rangle$ *all exist* $\rightarrow$ **Many-World interpretation**
2. Add new variables that pick a single branch $\rightarrow$ **Bohmian Mechanics**
3. Make the dynamics non-linear to pick a branch $\rightarrow$ **Collapse Models**







Everett 1957

Bohm 1952

Ghirardi 1986

The idea of objective collapse models

Schrödinger equation + non-linear peanut

$$\frac{\mathrm{d}}{\mathrm{d}t}\psi_t = -\frac{i}{\hbar}H\,\psi_t + \varepsilon(\psi)\ ,$$

H is the fundamental Hamiltonian or an approximation

Completely *ad hoc*, the objective is to show it is *possible*

The model of Ghirardi, Rimini, and Weber

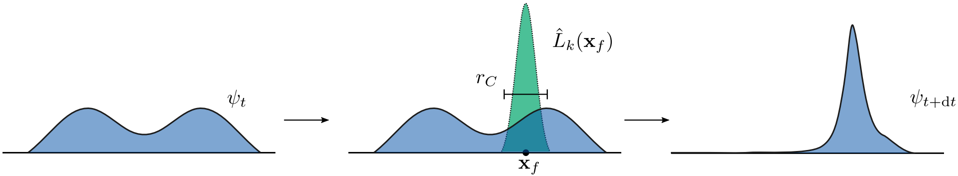
The GRW modification (1986)

Every dt, with proba λdt each particle collapses around a random x_f

$$\psi_t \longrightarrow \frac{\hat{L}(x_f)\psi_t}{\|\hat{L}(x_f)\psi_t\|} \quad \text{with proba} \quad P(x_f) = \|\hat{L}(x_f)\psi_t\|^2$$

with an envelope $\hat{L}(x_f) = \frac{1}{(\pi r_C^2)^{3/4}} e^{-(\hat{x}_k - x_f)^2/(2r_C^2)}$.

GianCarlo Ghirardi
1935 - 2018



Why it works

Fixing $\lambda = 10^{-16}\mathrm{s}^{-1}$ (historical value) :

1. An electron collapses every 300 million years.
2. A cat with $\simeq 10^{28}$ electrons, is localized up to r_c in less than a picosecond

Why it works

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Microscopic degrees of freedom do not collapse because of their intrinsic dynamics, but when they are coupled to something macroscopic (detector, human, etc.) $\implies$ Heisenberg cut objectified $\implies$ it is a theory of the world

Some wrong things about QT

Knowing interpretations / reconstructions *exist* for the black box helps dispel myths

Examples

1. "*Quantum mechanics shows the world is irreducibly random*"

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   $\rightarrow$ Bohmian mechanics is deterministic, randomness comes from our lack of knowledge of initial conditions

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2. "*Quantum mechanics show we have to profoundly rethink metaphysics and classical preconceptions of reality*"

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Examples

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   → Bohmian mechanics is deterministic, randomness comes from our lack of knowledge of initial conditions

2. "*Quantum mechanics show we have to profoundly rethink metaphysics and classical preconceptions of reality*"
   → Bohmian mechanics is a simple theory of particles that move around

So what is actually **surprising practically** with quantum mechanics?

Quantum non-locality

The irreducibly weird thing

EPR argument

In 1935, Einstein - Podolsky - Rosen (EPR) considered the state of two atoms far away from each other (say a light-year away)

$$|\psi_{\text{EPR}}\rangle = \frac{1}{\sqrt{2}}\big(|\text{ground}\rangle_1 \otimes |\text{excited}\rangle_2 - |\text{excited}\rangle_1 \otimes |\text{ground}\rangle_2\big)$$

Measure 1, get the result "ground", know immediately that 2 is "excited"

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Measure 1, get the result “ground”, know immediately that 2 is “excited”

For EPR this means either:

1. There is some action at a distance, faster than light – **impossible!**
2. The result was predetermined, and thus $|\psi_{\text{EPR}}\rangle$ is not all there is

Bell's modification

In 1964, in "*On the EPR paradox*" Bell considered the same state

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but chose to measures randomly P_0, $P_{2\pi/3}$, or $P_{4\pi/3}$ on each side, where

$$P_\theta = |\theta\rangle\langle\theta| \text{ with } |\theta\rangle = \cos(\theta)\,|\mathrm{ground}\rangle + \sin(\theta)\,|\mathrm{excited}\rangle$$

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We additionally flip the result by convention on one side, then

- If measurements are the same on each side: **perfect agreement** (EPR)
- If measurements are different: agreement with probability $\cos^2(2\pi/3) = 1/4$

Bell's theorem

NOT ABOUT QUANTUM MECHANICS

Simplified gamified version

Setup

- Take 2 players far away, who could establish strategies before but are now too far away to influence each other (locality).
- Consider 3 different questions, to which they can answer by `YES` or `NO`.

Hypothesis

- If the question they are asked is the same, they have to answer the same (both `YES`, or both `NO`).

Then

The probability that they answer the same thing when asked random **different** questions is necessarily greater than $1/3$ [Bell's inequality]

Bell's inequality is violated by Quantum Mechanics

If two players have one half of an EPR pair $|\psi_{\mathsf{EPR}}\rangle$ in a box they can

- Associate to each question a measurement P_θ for $\theta = 0, 2\pi/3, 4\pi/3$
- Answer `YES` or `NO` depending on the result they get

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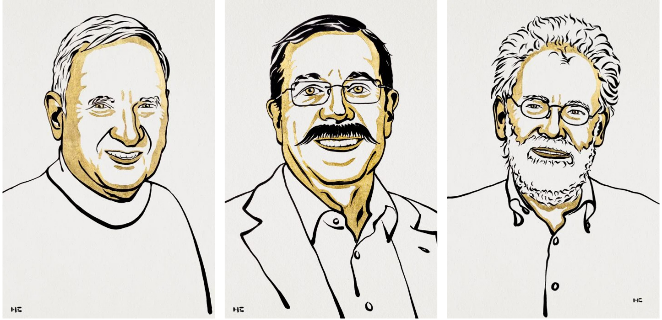
No faster than light signalling, but not just correlations

Entanglement does not allow the players to communicate, but allows them to collaborate in ways they couldn't without influencing each other.

The world violates Bell's inequality just like Quantum Mechanics predicts!

Nobel Prize 2022



The seed of the second quantum revolution (cryptography, computing)

Towards FTL-HFT?

A peculiar non-local auction

1 of 3 stocks A, B, C will be randomly traded in Paris and NYC **for only 1ms**.
[For example, A is traded in Paris, B in NYC, for 1ms]

Rules:

1. Having both of the same stock, long or short, is the best possible thing
   (+A+A, -B-B, $\cdots$) $\rightarrow +10$ points
2. Having one short one long of different stocks is second best
   (+A-B, +B-C, $\cdots$) $\rightarrow +0$ points
3. Having two longs / two shorts of different stocks is worst
   (+A+B, -C-B, $\cdots$) $\rightarrow -1$ points

Result:

- Best strategy with standard servers and optical fibers: 2.83
- Best strategy with a quantum memory in a cryostat and optical fibers: 2.89