Beating the quantum many-body problem

on the lattice, in the continuum,
with bits and qubits

DALL·E "Two cats falling into a blackhole in synthwave style, digital art"

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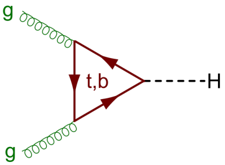
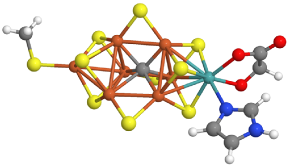
Open problems in theoretical physics

Fundamental Physics



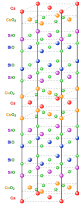
Strong force between quarks and gluons

Chemistry



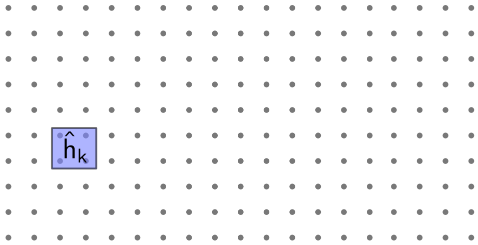
Atoms interacting to form molecules

Condensed matter



Cuprate perovskites superconductors

Quantum many-body problem on the lattice



Typical many-body problem

N spins on a lattice

$\mathscr{H} = \bigotimes_{j=1}^{n} \mathscr{H}_j$ with $\mathscr{H}_j = \mathbb{C}^2$

$|\psi\rangle = \sum c_{i_1, i_2, \cdots, i_n} |i_1, i_2, \cdots i_N\rangle$

Problem:

Finding the low energy states of

$$H = \sum_{k=1}^{N} h_k$$

is **hard** because $\dim \mathscr{H} = 2^N$ for spins

Fugaku – 2 EFLOPS – 150 PB
cannot do $4 \times 4 \times 4$ spins

Quantum-many body problem: bits or qubits?

With bits – weak entanglement $\rightarrow$ compression with TNS

- Local H
- Low energy state properties
- Dynamics after local quench

With qubits – strong entanglement $\rightarrow$ "exact" representation

- Unstructured H
- Highly excited state statistics
- Dynamics after global quench

With bits, and for fundamental physics

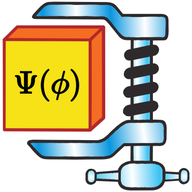
Two difficulties to solve

1. Lattice $\rightarrow$ continuum
2. Weird constrained degrees of freedom ("Gauge theories")
3. 3 space dimensions

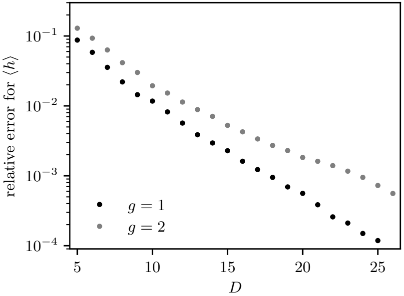
Tensor network states without lattice

Solve baby fundamental physics theories like

$$H = \int \mathrm{d}x \, \frac{:\pi^2:_m}{2} + \frac{:(\nabla\phi)^2:_m}{2} + \frac{m^2}{2} :\phi^2:_m + g :\phi^4:_m$$



continuously infinitely many bosons on a line, interacting



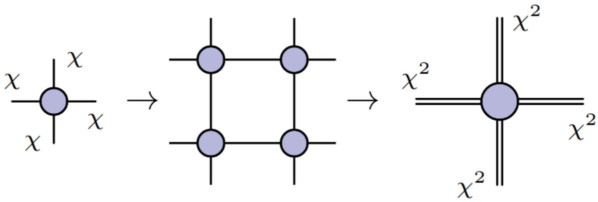
Karan and Edo: pushing to more models, beyond baby theories

Objective: completely nail down $1+1$ dimension

- $H_{\mathrm{ShG}}(\beta) = \int \mathrm{d}x \, \frac{:\pi^2:}{2} + \frac{:(\nabla\phi)^2:}{2} + \frac{m^2}{\beta^2} : \cosh(\beta\phi) :$
- $H_{\mathrm{SG}}(\beta) = \int \mathrm{d}x \, \frac{:\pi^2:}{2} + \frac{:(\nabla\phi)^2:}{2} - \frac{m^2}{\beta^2} : \cos(\beta\phi) :$
- $H_{O(2)} = \int \mathrm{d}x \, \frac{:|\pi^2|:}{2} + \frac{:|\nabla\phi|^2:}{2} + \frac{m^2}{2} : |\phi|^2 : + g : |\phi|^4 :_m$
- ...

Tensor network renormalization

TNS can also be used for powerful coarse graining / renormalization



Alternative to analytical continuum limit

- Étienne works on rigorous aspects of this approach
- Leonardo works on implementing gauge degrees of freedom