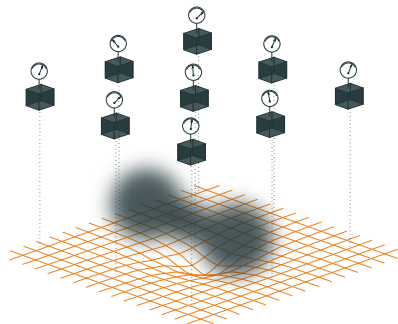# The general structure of quantum-classical dynamics

for gravity and more...

Antoine Tilloy
September 25th, 2024
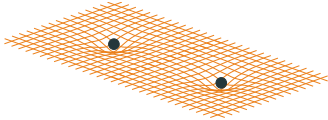Vienna Quantum Foundations
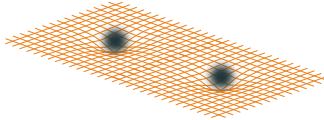Conference

Prolegomena

Classical gravity

- ▶ **Matter** is classical
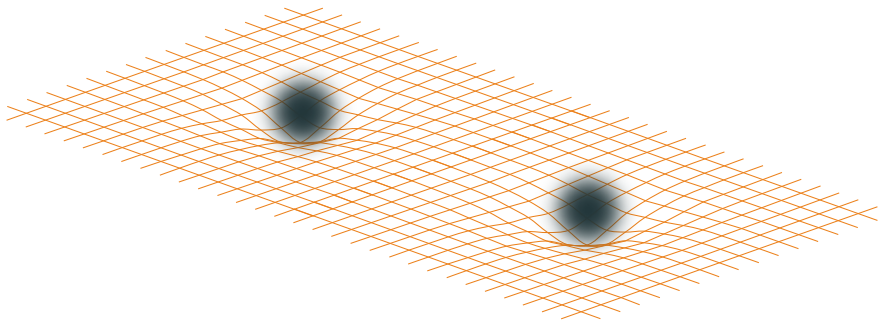- ▶ Spacetime is classical

Semiclassical gravity

- ▶ **Matter** is quantum
- ▶ Spacetime is classical

Fully quantum gravity

- ▶ **Matter** is quantum
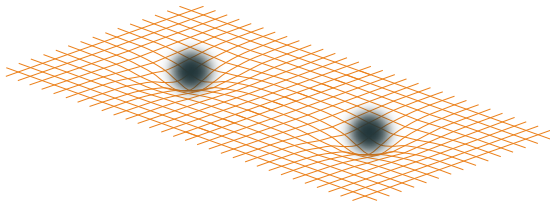- ▶ Spacetime is quantum

Standard semiclassical gravity

"Standard" semi-classical gravity

A semi-classical theory of gravity tells 2 stories:

1. Quantum matter moves in a curved classical space-time

2. The classical space time is curved by quantum matter

"Standard" semi-classical gravity

A semi-classical theory of gravity tells 2 stories:

  1. Quantum matter moves in a curved classical space-time

  2. The classical space time is curved by quantum matter
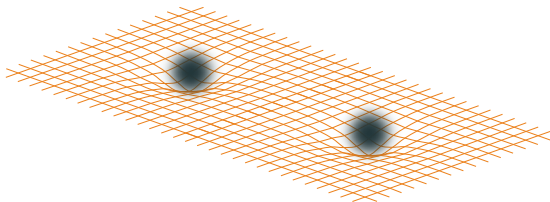
1 is known (QFTCST), **2** is not

> **The crucial question of semi-classical gravity is to know how quantum matter should source curvature.**

Møller-Rosenfeld semi-classical gravity

The **CHOICE** of Møller and Rosenfeld it to take:

$$R_{\mu\nu} - \frac{1}{2}R\,g_{\mu\nu} = 8\pi G\,\langle \hat{T}_{\mu\nu}\rangle$$

$\rightarrow$ source gravity via expectation values

Møller-Rosenfeld semi-classical gravity

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$$R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} = 8\pi G \, \langle \hat{T}_{\mu\nu} \rangle$$

$\rightarrow$ source gravity via expectation values

There are:

▶ **technical relativistic** difficulties [renormalization of $\langle T_{\mu\nu} \rangle$]

▶ **conceptual non-relativistic** difficulties [Born rule,$\cdots$].

Christian Møller

Leon Rosenfeld

Schrödinger-Newton

1. Non-relativistic limit of the "sourcing" equation:

$$\nabla^2 \Phi(x, t) = 4\pi G \langle \psi_t | \hat{M}(x) | \psi_t \rangle$$

Schrödinger-Newton

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$$\nabla^2 \Phi(x, t) = 4\pi G \langle \psi_t | \hat{M}(x) | \psi_t \rangle$$

2. Non-relativistic limit of QFTCST (just external field)

$$\frac{\mathrm{d}}{\mathrm{d}t} |\psi\rangle = -i \left(H_0 + \int \mathrm{d}x\, \Phi(x, t) \hat{M}(x) \right) |\psi_t\rangle,$$

Schrödinger-Newton

1. Non-relativistic limit of the "sourcing" equation:

$$\nabla^2 \Phi(x, t) = 4\pi G \langle \psi_t | \hat{M}(x) | \psi_t \rangle$$

2. Non-relativistic limit of QFTCST (just external field)

$$\frac{d}{dt} | \psi \rangle = -i \left(H_0 + \int dx \, \Phi(x, t) \hat{M}(x) \right) | \psi_t \rangle,$$

Putting the two together: Schrödinger-Newton equation

$$\frac{d}{dt} | \psi_t \rangle = -i H_0 | \psi_t \rangle + i \, G \int dx \, dy \, \frac{\langle \psi_t | \hat{M}(x) | \psi_t \rangle \, \hat{M}(y)}{|x - y|} | \psi_t \rangle.$$

The problem with deterministic non-linear theories

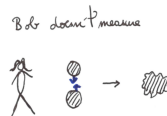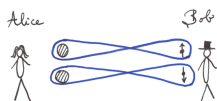Gisin 1989 $\rightarrow$ this is not allowed

Interpretations of QM are no longer empirically equivalent:

The problem with deterministic non-linear theories

Gisin 1989 $\rightarrow$ this is not allowed

Interpretations of QM are no longer empirically equivalent:

▶ **with collapse** (Copenhagen, GRW,...) – faster-than-light signalling

The problem with deterministic non-linear theories

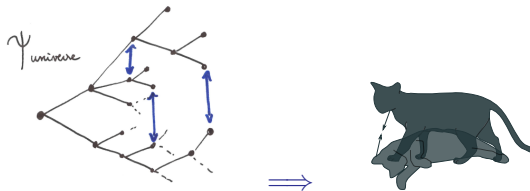Gisin 1989 $\rightarrow$ this is not allowed

Interpretations of QM are no longer empirically equivalent:

▶ **with collapse** (Copenhagen, GRW,...) – faster-than-light signalling

▶ **without collapse** (Many-Worlds, Bohm,...) – empirically absurd

The big question

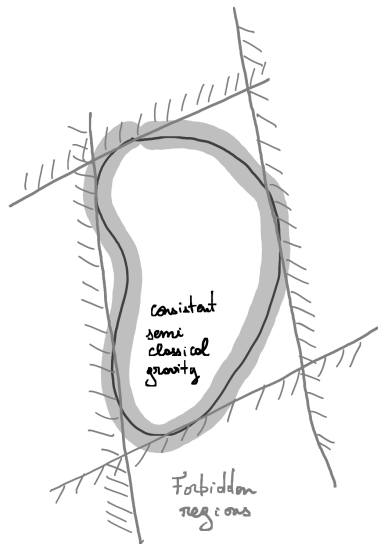> What mathematical object can one
> construct to source the gravitational field
> while keeping things consistent?

The big question, generalized

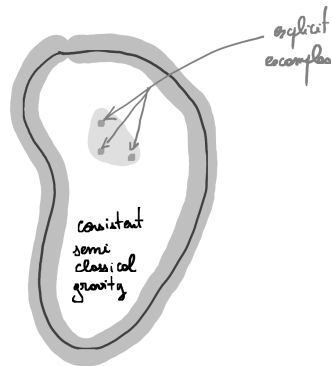How can one consistently couple
quantum and classical variables?

$$\rho_t \quad \longleftrightarrow \quad z_t$$

Two strategies to study the quantum nature of gravity

Bose et al., Marletto & Vedral   Kafri-Taylor-Milburn, Diósi, Oppenheim

3 ways to do this

1. Continuous measurement and feedback
2. Spontaneous collapse models
3. General continuous dynamics with a classical subspace [used by Oppenheim]

Main result from 2403.19748 → Scipost: the 3 are **mathematically equivalent**

General quantum-classical dynamics as measurement based feedback

Antoine Tilloy*

Laboratoire de Physique de l'Ecole Normale Supérieure, Mines Paris - PSL, CNRS, Inria, PSL Research University, Paris, France

Abstract

This note derives the stochastic differential equations and partial differential equation of general hybrid quantum-classical dynamics from the theory of continuous measurement and general (non-Markovian) feedback. The advantage of this approach is an explicit parameterization, without additional positivity constraints. The construction also neatly separates the different effects: how the quantum influences the classical and how the classical influences the quantum. This modular presentation gives a better intuition of what to expect from hybrid dynamics, especially when used to construct possibly fundamental theories.

The First Way: Continuous measurement and feedback

Continuous quantum measurement – derivation

Continuous measurement – without Zeno effect
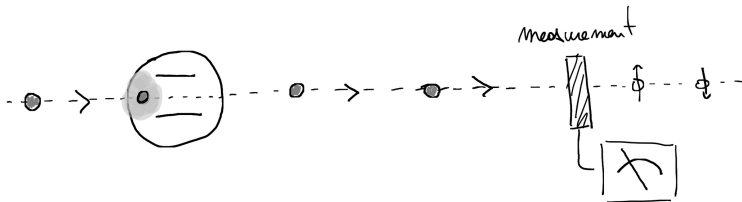
- time between ancillas $\Delta t \propto \epsilon$
- interaction strength $\omega \propto \sqrt{\epsilon}$

Continuous quantum measurement

Stochastic Master Equation ($\sim$ **1987 – pre-theory by Gisin 1984)**

Density matrix:

$$\mathrm{d}\rho_t = \underbrace{-i[H, \rho_t]\,\mathrm{d}t}_{\text{standard quantum dynamics}} + \underbrace{\mathcal{D}[\hat{c}](\rho_t)\,\mathrm{d}t}_{\text{decoherence}} + \underbrace{\mathcal{H}[\hat{c}](\rho_t)\,\mathrm{d}W_t}_{\text{measurement backaction}}$$

Signal:

$$\mathrm{d}r_t = \mathrm{tr}\left[(\hat{c} + \hat{c}^\dagger)\,\rho_t\right]\,\mathrm{d}t + \mathrm{d}W_t$$

with:

- $\mathcal{D}[\mathcal{O}](\rho) = \mathcal{O}\rho\mathcal{O}^\dagger - \frac{1}{2}\left(\mathcal{O}^\dagger\mathcal{O}\rho + \rho\mathcal{O}^\dagger\mathcal{O}\right)$
- $\mathcal{H}[\mathcal{O}](\rho) = \mathcal{O}\rho + \rho\mathcal{O}^\dagger - \mathrm{tr}\left[(\mathcal{O} + \mathcal{O}^\dagger)\,\rho\right]\rho$
- $\frac{\mathrm{d}W_t}{\mathrm{d}t}$ "white noise"

V. Belavkin

A. Barchielli

L. Diósi

The measurement signal

The signal – or continuous result

$$\mathrm{d}r_t = \mathrm{tr}\left[\left(\hat{c} + \hat{c}^\dagger\right)\rho_t\right]\mathrm{d}t + \mathrm{d}W_t$$

$\rightarrow$ A noisy version of the quantum expectation value

Experimental aside

The signal y_t is routinely measured, for various measured operators, in superconducting circuits using via homodyne / heterodyne detection
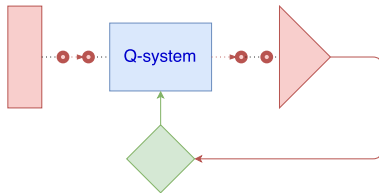
Measurement based feedback
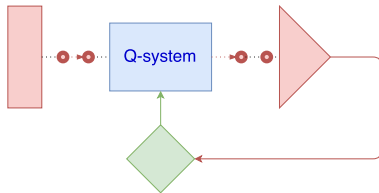
Step 1: Have the classical z_t depend on r_t

$$dz_t = F(z_t)\, dt + G(z_t)\, dr_t$$

Step 2: Have the quantum depend on the classical

$$H \longrightarrow H + V(z_t)$$

Measurement based feedback

Step 1: Have the classical z_t depend on r_t

$$\mathrm{d}z_t = F(z_t)\,\mathrm{d}t + G(z_t)\,\mathrm{d}r_t$$

Step 2: Have the quantum depend on the classical

$$H \longrightarrow H + V(z_t)$$

Consistent by construction since derivable as *effective* from Copenhaguen QM

Most general stochastic equations

Quantum stochastic master equation

$$d\rho = -i[H_0 + V(\mathbf{z}), \rho]\, dt + \sum_{k=1}^{n} \mathcal{D}[\hat{c}_k](\rho)\, dt + \sqrt{\eta_k}\, \mathcal{M}[\hat{c}_k](\rho)\, dW_k,$$

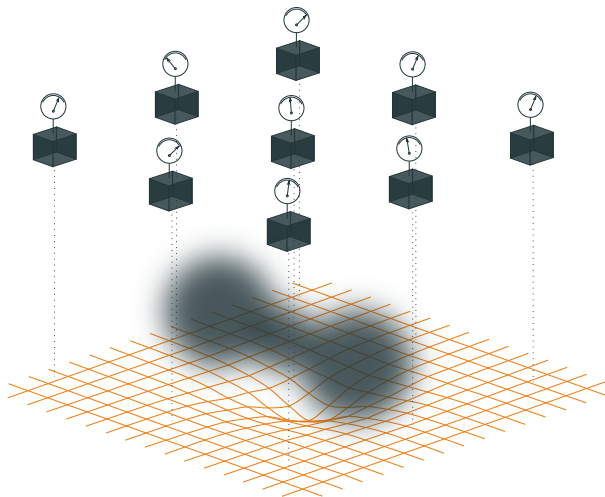**Signal stochastic differential equation** (the quantum classical glue)

$$dr_k = \frac{1}{2}\mathrm{tr}[(\hat{c}_k + \hat{c}_k^\dagger)\rho]\, dt + \frac{1}{2\sqrt{\eta_k}}dW_k.$$

Classical stochastic differential equation

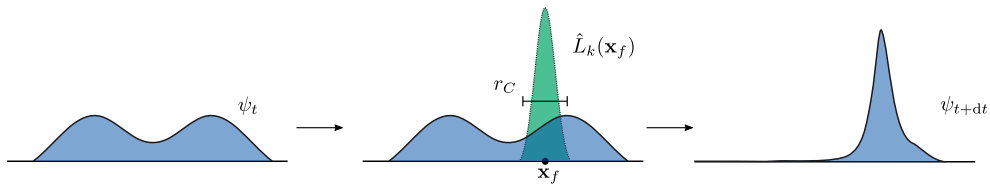$$dz_a = F_a(\mathbf{z})\, dt + G_{ak}(\mathbf{z})\,\sqrt{\eta_k}\, dr_k$$

F, G are functions – $V(\mathbf{z})$, $\hat{c}_k$ are operators

"Intuition pump" picture for gravity

AS IF – "There are detectors in space-time measuring the mass density continuously and curving space-time accordingly." $\rightarrow$ explains consistency

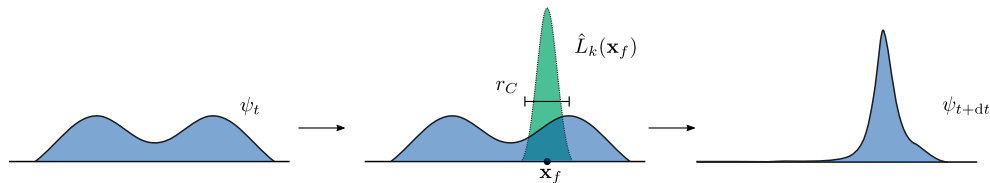# The Second Way: Spontaneous collapse models

The idea of collapse models

Other names: [Objective / spontaneous / dynamical] [reduction / collapse] [model / program]

Schödinger equation + tiny non-linear bit

$$\frac{\mathrm{d}}{\mathrm{d}t}\psi_t = -\frac{i}{\hbar}H\psi_t + \varepsilon(\psi)\,,$$

H is the Standard Model Hamiltonian (or non-relativistic approx)

The easier Ghirardi-Rimini-Weber model
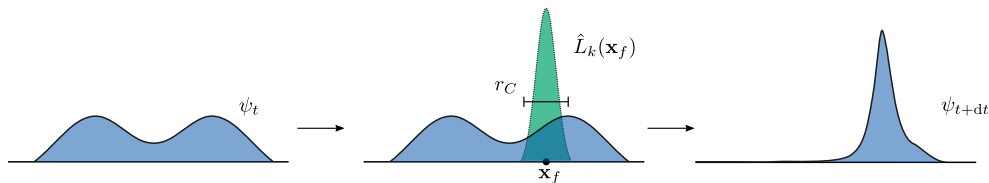
The GRW modification (1986)

Every dt, with probability λdt particle k collapses around point x_f

$$\psi_t \longrightarrow \frac{\hat{L}_k(x_f)\psi_t}{\|\hat{L}_k(x_f)\psi_t\|} \text{ with proba } P(x_f) = \|\hat{L}_k(x_f)\psi_t\|^2$$

with an envelope $\hat{L}_k(x_f) = \frac{1}{(\pi r_C^2)^{3/4}} e^{-(\hat{x}_k-x_f)^2/(2r_C^2)}$.

GianCarlo Ghirardi
1935 - 2018

Spontaneous collapse models

Mathematically, (continuous) Markovian spontaneous collapse models are equivalent to continuous measurement of appropriate observables

$$\mathcal{O} \longrightarrow \hat{M}_\sigma(x)$$

Metaphysics – Ontology – beables

What is real ? What is the world made of ?

Metaphysics – Ontology – beables

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1. GRW0 The wave-function ψ_t itself (but infinite literature of subtleties)

Metaphysics – Ontology – beables

What is real ? What is the world made of ?

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2. GRWm The mass density $\langle \hat{M}(x) \rangle$

$$\langle \hat{M}(x) \rangle = \sum_k \int dx_1 \cdots dx_n \underbrace{|\psi(x_1, \cdots, x, \cdots, x_n)|^2}_{x \text{ in } k^{\text{th}} \text{ position}}$$

Metaphysics – Ontology – beables

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3. GRWf The events (t_f, x_f) where the wave-function collapse (the flashes) – [Bell's choice!]
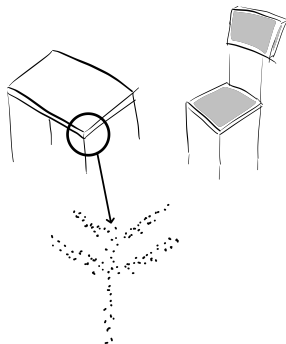
Metaphysics – Ontology – beables
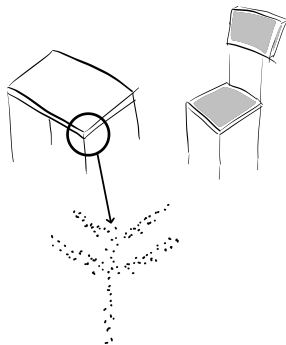
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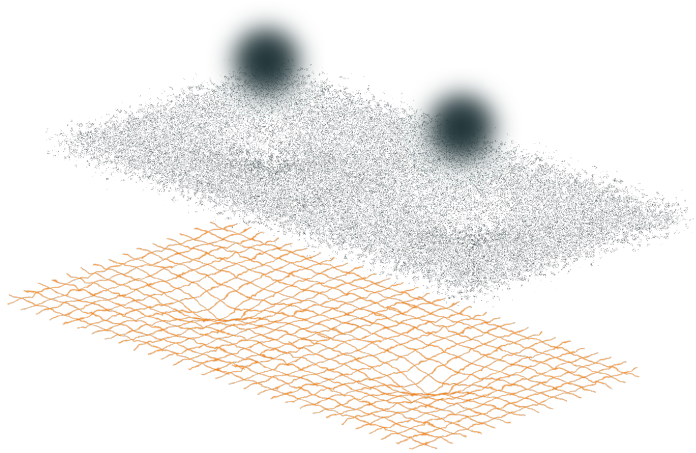
3. GRWf The events (t_f, x_f) where the wave-function collapse (the flashes) – [Bell's choice!]

Fact: (continuous) flashes $=$ signal

Collapse model picture of hybrid dynamics

"The gravitational interaction is mediated by a stochastic field, which is the **local beable** of the theory"

The Third Way: Embedding a classical sector in quantum dynamics

$$\rho(z, t)$$

Formulation of the problem

Quantum-classical state

A state diagonal in the classical variables z

$$\rho_{QC} = \int dz \, \rho_Q(z) \, |z\rangle\langle z|$$

► used early on (Diosi, Halliwell, Gisin)
► starting point of Oppenheim

A Postquantum Theory of Classical Gravity?

Jonathan Oppenheim
*Department of Physics and Astronomy, University College London,
Gower Street, London WC1E 6BT, United Kingdom*

The effort to discover a quantum theory of gravity is motivated by the need to reconcile the incompatibility between quantum theory and general relativity. Here, we present an alternative approach by constructing a consistent theory of classical gravity coupled to quantum field theory. The dynamics is linear in the density matrix, completely positive, and trace preserving, and reduces to Einstein's theory of general relativity in the classical limit. Consequently, the dynamics does not suffer from the pathologies of the semiclassical theory based on expectation values. The assumption that general relativity is classical necessarily modifies the dynamical laws of quantum mechanics: the theory must be fundamentally stochastic in both the metric degrees of freedom and in the quantum matter fields. This breakdown in predictability allows it to evade several no-go theorems purporting to forbid classical quantum interactions. The measurement postulate of quantum mechanics is not needed; the interaction of the quantum degrees of freedom with classical space-time necessarily causes decoherence in the quantum system. We first derive the general form of classical quantum dynamics and consider realizations which have as its limit deterministic classical Hamiltonian evolution. The formalism is then applied to quantum field theory interacting with the classical space-time metric. One can view the classical quantum theory as fundamental or as an effective theory useful for computing the backreaction of quantum fields on geometry. We discuss a number of open questions from the perspective of both viewpoints.

The Physicist Who's Challenging the Quantum Orthodoxy

For decades, physicists have struggled to develop a quantum theory of gravity. But what if gravity — and space-time — are fundamentally classical?

Most general second order PDE

Constraints:

- Assuming z evolves continuously $\rightarrow$ at most second order derivatives
- $\rho(z)$ physical $\rightarrow$ positivity conditions

$$
\begin{aligned}
\frac{\partial \rho_t(\mathbf{z})}{\partial t} = &- i\left[H, \rho_t(\mathbf{z})\right] + \sum_{k=1}^{n} \mathcal{D}[\hat{c}_k](\rho_t(\mathbf{z})) \\
&- \frac{\partial}{\partial z_a}\left[F_a(\mathbf{z})\rho_t(\mathbf{z}) + \frac{\sqrt{\eta_k}\,G_{ak}(\mathbf{z})}{2}\left(\hat{c}_k\rho_t(\mathbf{z}) + \rho_t(\mathbf{z})\hat{c}_k^\dagger\right)\right] \\
&+ \frac{1}{2}\frac{\partial^2}{\partial z_a \partial z_b}\left[\frac{G_{ak}(\mathbf{z})G_{bk}(\mathbf{z})}{4}\rho_t(\mathbf{z})\right]
\end{aligned}
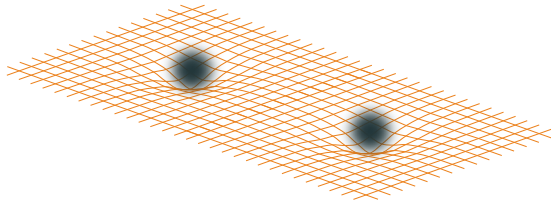$$

Equivalence via Ito's lemma

This PDE is the "**Fokker-Planck**" version of the "**Langevin**" dynamics of measurement + feedback:

Equivalence

Using Itô's lemma, one has:

$$\forall f \qquad \underset{\text{measurement and feedback}}{\mathbb{E}[\rho_t\, f(\mathbf{z}_t)]} = \underset{\text{hybrid PDE}}{\int f(\mathbf{z})\, \rho_t(\mathbf{z})\, d\mathbf{z}}\,.$$

Back to gravity

History

Newtonian early work

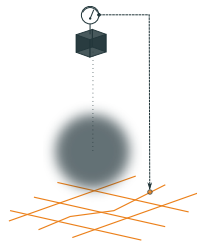Source gravity by measuring the mass density:

$$\nabla^2 \Phi(x) = 4\pi G \mathscr{S}_{\hat{M}}(x)$$

toy model – [Kafri, Taylor, Milburn 2014]
full Newtonian potential – [Diósi & T 2015]

General relativistic extensions

Construct a PDE for $\rho(z)$ for z the gravitational degrees of freedom in ADM general relativity

[Oppenheim, Weller-Davies, Layton, Soda, Russo, ... 2018 $\to$ today]

Markovian/Newtonian limit

Technically

Newtonian limit $=$ Markovian feedback limit

$$z \propto \mathrm{d}r$$

$\implies$ technically infinitely easier $\implies$ one can say something

Experimentally

Hard to probe anything else in the near future

Model

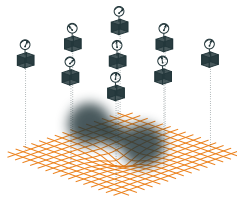1. **Step 1: continuous mass density measurement**
   We **imagine** that space-time is filled with detectors weakly measuring the mass density:

   The equation for matter is now as before with

   $\mathcal{O} \to \hat{M}(x), \ \forall x \in \mathbb{R}^3$

   $\gamma \to \gamma(x, y)$ coding detector strength and correlation

   and there is a "mass density signal" $S(x)$ in every point.

Model

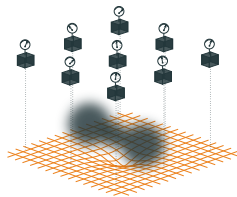1. **Step 1: continuous mass density measurement**
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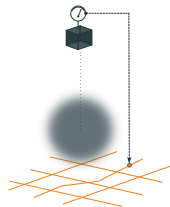   and there is a "mass density signal" $S(x)$ in every point.

2. **Step 2: Feedback**
   We take the mass density signal $S(x)$ to source the gravitational field φ:

   $$\nabla^2 \varphi(x) = 4\pi \, G \, S(x)$$

   which is **formally** equivalent to quantum feedback.

Result

Standard quantum feedback like computations give for $\rho_t = \mathbb{E}[|\psi_t\rangle\langle\psi_t|]$:

$$\partial_t \rho = -i\left[H_0 + \frac{1}{2}\iint dx dy \, \mathscr{V}(x,y)\hat{M}(x)\hat{M}(y), \rho_t\right]$$
$$- \frac{1}{8}\iint dx dy \, \mathscr{D}(x,y)\Big[\hat{M}(x), \big[\hat{M}(y), \rho_t\big]\Big],$$

with the **gravitational pair-potential**

$$\mathscr{V} = \left[\frac{4\pi G}{\nabla^2}\right](x,y) = -\frac{G}{|x-y|},$$

and the **positional decoherence**

$$\mathscr{D}(x,y) = \left[\frac{\gamma}{4} + \mathscr{V} \circ \gamma^{-1} \circ \mathscr{V}^\top\right](x,y)$$

Principle of least decoherence

$$\mathscr{D}(x,y) = \left[\frac{\gamma}{4} + \mathscr{V} \circ \gamma^{-1} \circ \mathscr{V}^{\top} \right](x,y)$$

There is still a (functional) degree of freedom $\gamma(x,y)$:

▶ Large $\|\gamma\| \implies$ strong "measurement" induced decoherence
▶ Small $\|\gamma\| \implies$ strong "feedback" decoherence

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There is an optimal (non-local) kernel that minimizes decoherence.

Principle of least decoherence

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There is an optimal (non-local) kernel that minimizes decoherence.

Diagonalizing in Fourier, one gets a global minimum for

$$\gamma = 2\sqrt{\mathcal{V} \circ \mathcal{V}^\top} = -2\mathcal{V}$$

Hence:

$$\mathcal{D}(x, y) = -\mathcal{V}(x, y) = \frac{G}{|x - y|}$$

This is just the decoherence kernel of the Diósi-Penrose model (erstwhile heuristically derived)!

Regularization

Even for the minimal decoherence prescription, the decoherence is **infinite**.

Regularization

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Adding a regulator at a length scale σ has 2 effects:

- ▶ It tames decoherence, making it finite
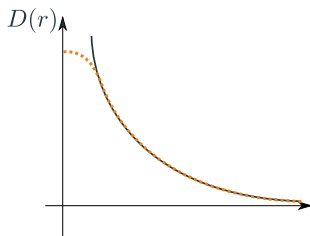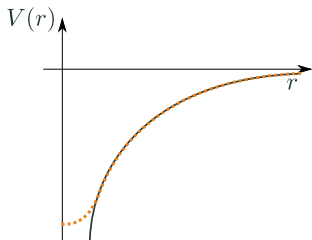- ▶ It regularizes the pair potential $\propto \frac{1}{r}$ for $r \lesssim \sigma$

Regularization

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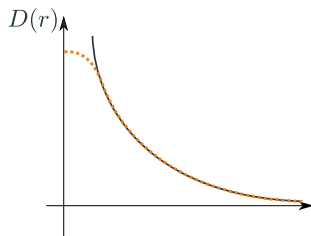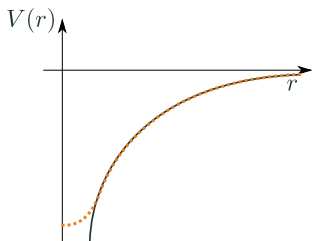- $\implies$ there is a **trade-off**.

Regularization

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$\implies$ there is a **trade-off**.

Experimentally:

$$\underbrace{10^{-15} \to 10^{-10} m}_{\text{decoherence constraint}} \ll \sigma \leqslant \underbrace{10^{-4} m}_{\text{gravitational constraint}}$$

Importantly $\sigma > \ell_{\text{Compton}} \gg \ell_{\text{Planck}}$.

Summary

Conceptually: 3 equivalent ways to construct hybrid quantum-classical dynamics

1. **Measurement and feedback** which shows **consistency**
2. **Spontaneous collapse** which shows **empirical effects** + **measurement problem** solution
3. **Quantum Classical PDE** which shows **generality**

Quantum classical dynamics are *possible* and *well understood*

For gravity

▶ Newtonian limit: well defined models – minimizing decoherence gives Diosi-Penrose model

▶ General case: being explored by Oppenheim et al. – big progress, but not clear all constraints can be met