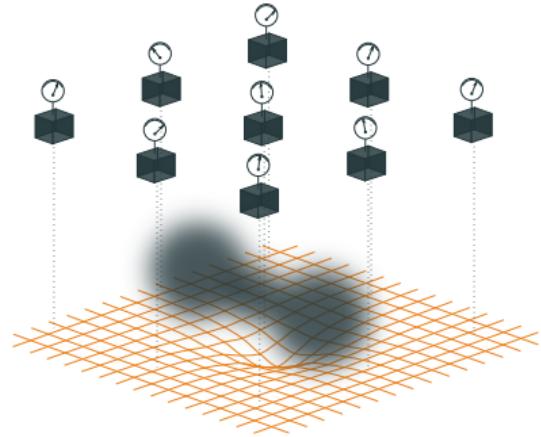


The general structure of quantum-classical dynamics

and implications for gravity and entanglement generation...

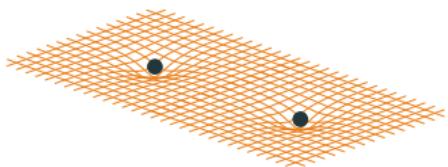


Antoine Tilloy
July 2nd, 2025
Foundations 2025
Gdansk, Poland



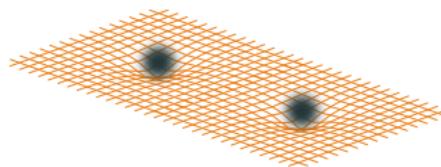
Prolegomena

Classical gravity



- ▶ **Matter** is classical
- ▶ **Spacetime** is classical

Semiclassical gravity



- ▶ **Matter** is quantum
- ▶ **Spacetime** is classical

Fully quantum gravity



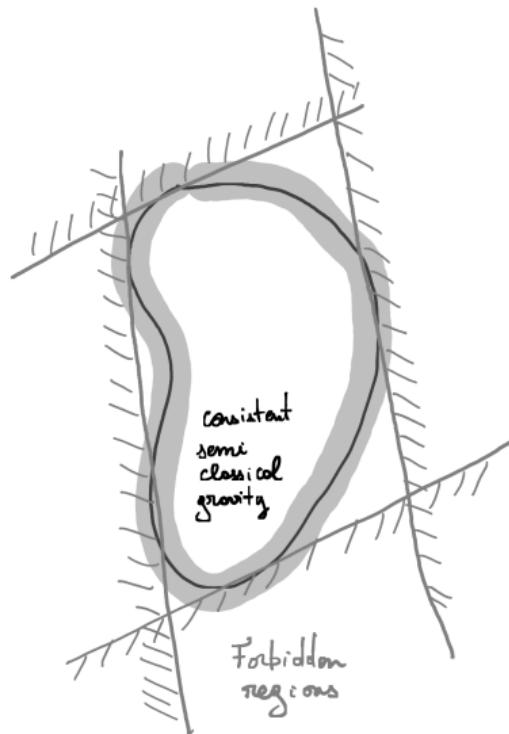
- ▶ **Matter** is quantum
- ▶ **Spacetime** is quantum

Is the chimera possible?

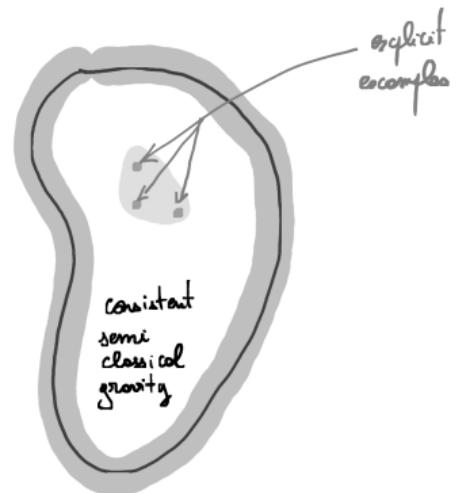


I bet 99 to one that the outcome [of some proposed experiments] will be consistent with gravity having quantum properties. – Carlo Rovelli

Two strategies to study the quantum nature of gravity

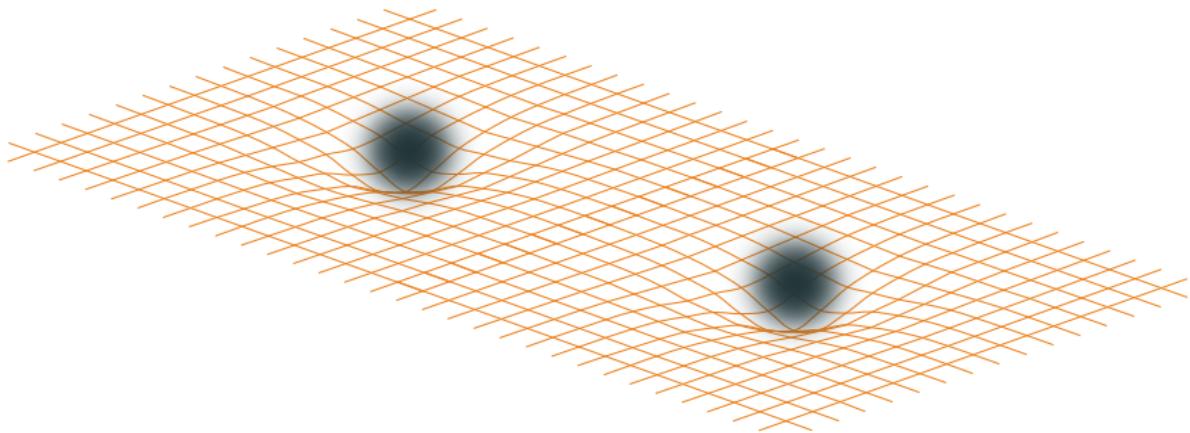


Bose et al., Marletto & Vedral



Kafri-Taylor-Milburn, Diósi, Oppenheim

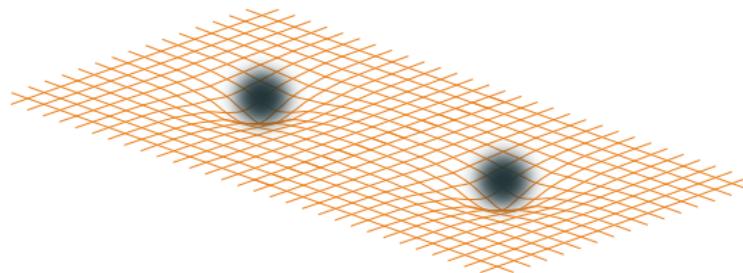
Standard semiclassical gravity



“Standard” semi-classical gravity

A semi-classical theory of gravity tells 2 stories:

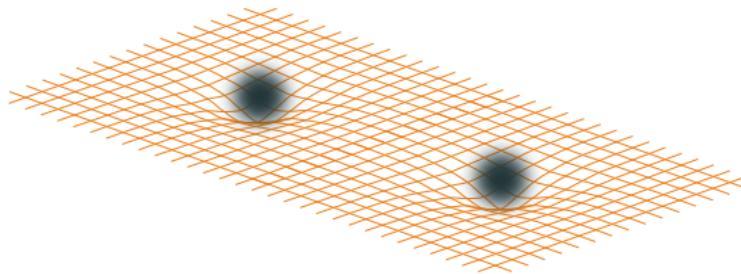
1. Quantum matter moves in a curved classical space-time
2. The classical space time is curved by quantum matter



“Standard” semi-classical gravity

A semi-classical theory of gravity tells 2 stories:

1. Quantum matter moves in a curved classical space-time
2. The classical space time is curved by quantum matter



1 is known (QFTCST), **2** is not

The crucial question of semi-classical gravity is to know how quantum matter should source curvature.

Møller-Rosenfeld semi-classical gravity

The **CHOICE** of Møller and Rosenfeld it to take:

$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} = 8\pi G \langle \hat{T}_{\mu\nu} \rangle$$

→ source gravity via expectation values

Møller-Rosenfeld semi-classical gravity

The **CHOICE** of Møller and Rosenfeld it to take:

$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} = 8\pi G \langle \hat{T}_{\mu\nu} \rangle$$

→ source gravity via expectation values

There are:

- ▶ **technical relativistic** difficulties [renormalization of $\langle T_{\mu\nu} \rangle$]
- ▶ **conceptual non-relativistic** difficulties [Born rule, ...].

$$\frac{d}{dt}|\psi_t\rangle = -iH_0|\psi_t\rangle + iG \int dx dy \frac{\langle \psi_t | \hat{M}(x) | \psi_t \rangle \hat{M}(y)}{|x-y|} |\psi_t\rangle.$$



Christian Møller



Leon Rosenfeld

The problem with deterministic non-linear theories

Gisin 1989 (and Diósi, Polchinsky) → this is not allowed

Interpretations of QM are no longer empirically equivalent:

The problem with deterministic non-linear theories

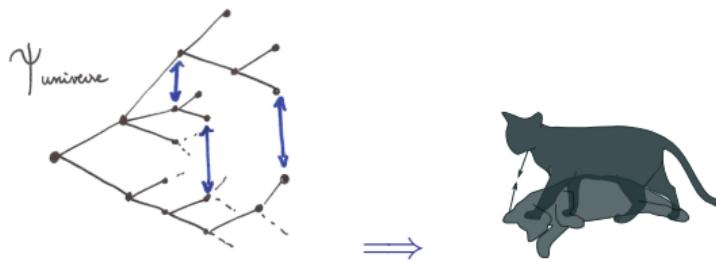
Gisin 1989 (and Diósi, Polchinsky) → this is not allowed

Interpretations of QM are no longer empirically equivalent:

- ▶ **with collapse** (Copenhagen, GRW,...) – faster-than-light signalling



- ▶ **without collapse** (Many-Worlds, Bohm,...) – empirically absurd



The big question

What mathematical object can one construct to source the gravitational field while keeping things consistent?

The big question, generalized

How can one consistently couple
quantum and classical variables?

$$\rho_t \longleftrightarrow z_t$$

3 ways to do this

1. Continuous measurement and feedback
2. Spontaneous collapse models
3. General continuous dynamics with a classical subspace [used by Oppenheim]

Main result from 2403.19748 → Scipost: the 3 are **mathematically equivalent**

General quantum-classical dynamics as measurement based
feedback

Antoine Tilloy*

*Laboratoire de Physique de l'Ecole Normale Supérieure, Mines Paris - PSL, CNRS, Inria, PSL Research
University, Paris, France*

1t-ph] 5 May 2024

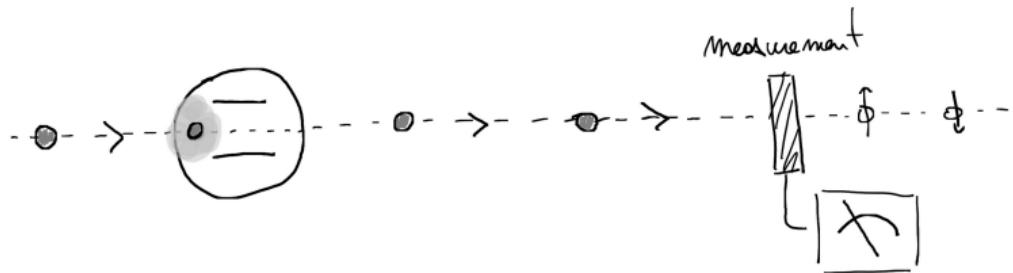
Abstract

This note derives the stochastic differential equations and partial differential equation of general hybrid quantum-classical dynamics from the theory of continuous measurement and general (non-Markovian) feedback. The advantage of this approach is an explicit parameterization, without additional positivity constraints. The construction also neatly separates the different effects: how the quantum influences the classical and how the classical influences the quantum. This modular presentation gives a better intuition of what to expect from hybrid dynamics, especially when used to construct possibly fundamental theories.

The First Way: Continuous measurement and feedback

Continuous quantum measurement – derivation

Continuous measurement – without Zeno effect



- ▶ time between ancillas $\Delta t \propto \varepsilon$
- ▶ interaction strength $\omega \propto \sqrt{\epsilon}$

Continuous quantum measurement

Stochastic Master Equation (~ 1987 – pre-theory by Gisin
1984)

Density matrix:

$$d\rho_t = \begin{array}{c} -i[H, \rho_t] dt \\ \text{standard quantum dynamics} \end{array} + \mathcal{D}[\hat{c}](\rho_t) dt + \mathcal{H}[\hat{c}](\rho_t) dW_t \begin{array}{c} \mathcal{D}[\hat{c}](\rho_t) dt \\ \text{decoherence} \end{array} \begin{array}{c} \mathcal{H}[\hat{c}](\rho_t) dW_t \\ \text{measurement backaction} \end{array}$$

Signal:

$$dr_t = \text{tr} \left[(\hat{c} + \hat{c}^\dagger) \rho_t \right] dt + dW_t$$

with:

- $\mathcal{D}[\mathcal{O}](\rho) = \mathcal{O}\rho\mathcal{O}^\dagger - \frac{1}{2} (\mathcal{O}^\dagger\mathcal{O}\rho + \rho\mathcal{O}^\dagger\mathcal{O})$
- $\mathcal{H}[\mathcal{O}](\rho) = \mathcal{O}\rho + \rho\mathcal{O}^\dagger - \text{tr} [(\mathcal{O} + \mathcal{O}^\dagger) \rho] \rho$
- $\frac{dW_t}{dt}$ “white noise”



V. Belavkin

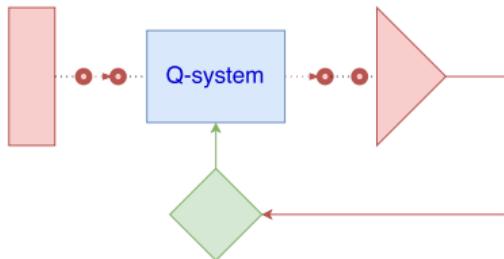


A. Barchielli



L. Diósi

Measurement based feedback



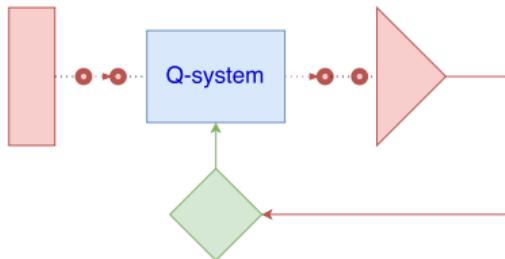
Step 1: Have the classical z_t depend on r_t

$$dz_t = F(z_t) dt + G(z_t) dr_t$$

Step 2: Have the quantum depend on the classical

$$H \longrightarrow H + V(z_t)$$

Measurement based feedback



Step 1: Have the classical z_t depend on r_t

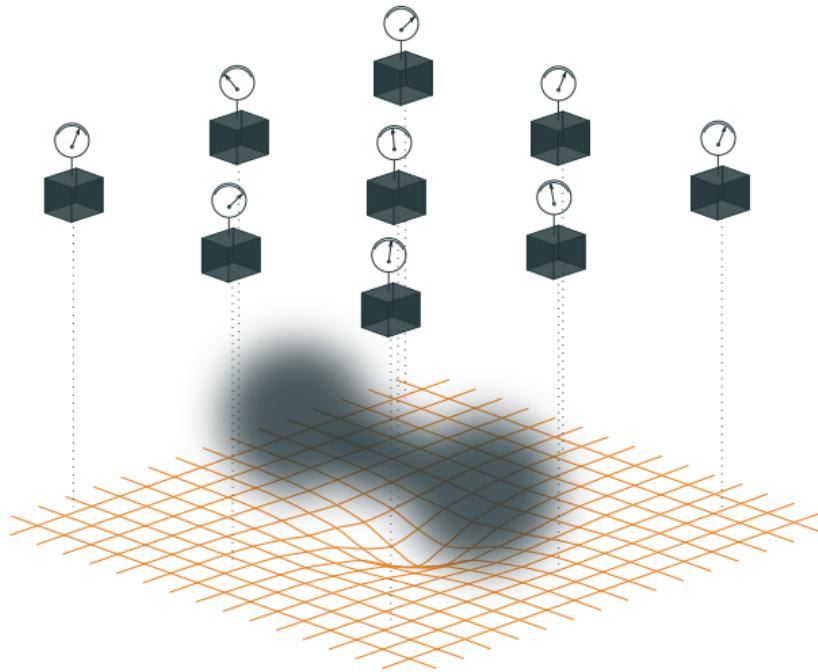
$$dz_t = F(z_t) dt + G(z_t) dr_t$$

Step 2: Have the quantum depend on the classical

$$H \longrightarrow H + V(z_t)$$

Consistent by construction since derivable as *effective* from Copenhagen QM

“Intuition pump” picture for gravity



AS IF – “There are detectors in space-time measuring the mass density continuously and curving space-time accordingly.” → explains consistency

The Third Way: Embedding a classical sector in quantum dynamics

$$\rho(z, t)$$

Formulation of the problem

Quantum-classical state

A state diagonal in the classical variables z

$$\rho_{QC} = \int dz \rho_Q(z) |z\rangle\langle z|$$

- ▶ used early on (Diosi, Halliwell, Gisin)
- ▶ starting point of Oppenheim

PHYSICAL REVIEW X 13, 041040 (2023)

Featured in Physics

A Postquantum Theory of Classical Gravity?

Jonathan Oppenheim¹

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Gower Street, London WC1E 6BT, United Kingdom

(Received 25 June 2021; revised 20 March 2023; accepted 5 October 2023; published 4 December 2023)

The effort to discover a quantum theory of gravity is motivated by the need to reconcile the incompatibility between quantum theory and general relativity. Here, we present an alternative approach by constructing a consistent theory of classical gravity coupled to quantum field theory. The dynamics is linear in the density matrix, completely positive, and trace preserving, and reduces to Einstein's theory of general relativity in the classical limit. Consequently, the dynamics does not suffer from the pathologies of the semiclassical theory based on expectation values. The assumption that general relativity is classical necessarily modifies the dynamical laws of quantum mechanics; the theory must be fundamentally stochastic in both the metric degrees of freedom and in the quantum matter fields. This breakdown in predictability allows it to evade several no-go theorems purporting to forbid classical quantum interactions. The measurement postulate of quantum mechanics is not needed: the interaction of the quantum degrees of freedom with classical space-time necessarily causes decoherence in the quantum system. We first derive the general form of classical quantum dynamics and consider realizations which have as its limit deterministic classical Hamiltonian evolution. The formalism is then applied to quantum field theory interacting with the classical space-time metric. One can view the classical quantum theory as fundamental or as an effective theory useful for computing the backreaction of quantum fields on geometry. We discuss a number of open questions from the perspective of both viewpoints.

DOI: 10.1103/PhysRevX.13.041040

Subject Areas: Gravitation,
Quantum Information

The Physicist Who's Challenging the Quantum Orthodoxy

...
...

For decades, physicists have struggled to develop a quantum theory of gravity. But what if gravity -- and space-time -- are fundamentally classical?



Most general second order PDE

Constraints:

- ▶ Assuming z evolves continuously \rightarrow at most second order derivatives

Most general second order PDE

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- ▶ $\rho(z)$ physical \rightarrow positivity conditions

Most general second order PDE

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- ▶ Assuming z evolves continuously \rightarrow at most second order derivatives
- ▶ $\rho(z)$ physical \rightarrow positivity conditions

$$\begin{aligned}\frac{\partial \rho_t(z)}{\partial t} = & -i[H, \rho_t(z)] + \sum_{k=1}^n \mathcal{D}[\hat{c}_k](\rho_t(z)) \\ & - \frac{\partial}{\partial z_a} \left[F_a(z) \rho_t(z) + \frac{\sqrt{\eta_k} G_{ak}(z)}{2} (\hat{c}_k \rho_t(z) + \rho_t(z) \hat{c}_k^\dagger) \right] \\ & + \frac{1}{2} \frac{\partial^2}{\partial z_a \partial z_b} \left[\frac{G_{ak}(z) G_{bk}(z)}{4} \rho_t(z) \right]\end{aligned}$$

Most general second order PDE

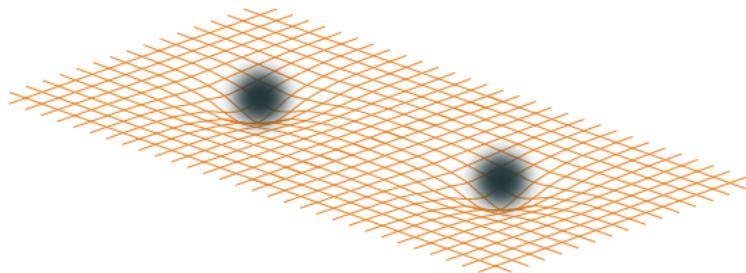
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- ▶ The “**Fokker-Planck**” version of the “**Langevin**” dynamics of measurement + feedback [proof via Itô’s lemma]

Back to gravity



Markovian/Newtonian limit

$$\Delta\Phi(x) = 4\pi GM(x)$$

Technically easier

Newtonian limit = Markovian feedback limit

classical variable at time $t \propto$ measurement signal at time t

\implies technically infinitely easier \implies one can say something

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Experimentally motivated

Hard to probe anything else in the near future **and unknown!**

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Experimentally motivated

Hard to probe anything else in the near future **and unknown!**

Can be nitpicked

No “independent” gravitational degrees of freedom. Can it be extended?

Concrete model

1. Step 1: continuous measurement

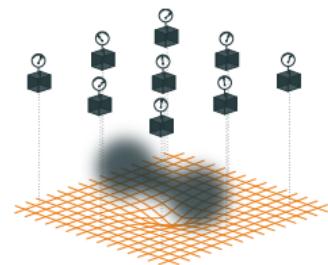
Choice of operators measure (infinitely many):

- ▶ Mass density [ultra local]

$$\mathcal{O} \rightarrow \hat{M}(x) \text{ then solve } \Delta\Phi(x) = 4\pi GM(x)$$

- ▶ Directly gravitational field [non-local, minimally decoherent]

$$\mathcal{O} \rightarrow \hat{\Phi}(x) = G \int d^3y \frac{\hat{M}(y)}{|x - y|}$$



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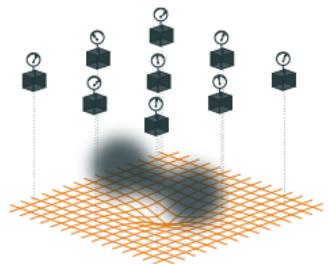
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2. Step 2: Feedback

Put classical field $\Phi(x)$ in Schrödinger equation



Result

Standard computations for $\hat{\Phi}(x)$ measurement and feedback give, for $\rho_t = \mathbb{E}[\psi_t\rangle\langle\psi_t]$:

$$\begin{aligned}\partial_t \rho = & -i \left[H_0 + \frac{1}{2} \iint dx dy \mathcal{V}(x, y) \hat{M}(x) \hat{M}(y), \rho_t \right] \\ & - \frac{1}{8} \iint dx dy \mathcal{D}(x, y) \left[\hat{M}(x), [\hat{M}(y), \rho_t] \right],\end{aligned}$$

with the **gravitational pair-potential**

$$\mathcal{V} = \left[\frac{4\pi G}{\nabla^2} \right] (x, y) = -\frac{G}{|x - y|},$$

and the **positional decoherence**

$$\mathcal{D}(x, y) = \frac{G}{|x - y|} \quad \text{Diósi-Penrose decoherence}$$

⇒ Diosi-Penrose model is the minimally decoherent semi-classical gravity model!

Lack of measurement locality and entanglement

Ability to generate entanglement as a smoking of **quantum gravity**:

- ▶ Bose et al. / Marletto & Vedral 2017

The minimally-decoherent option

$$\mathcal{O}(x) \rightarrow \hat{\Phi}(x) = G \int d^3y \frac{\hat{M}(y)}{|x - y|}$$

does generate entanglement because measurement *not spatially local*
[Trillo & Navascues 2024] [Feng, Marletto, Vedral 2025]

Quantitatively very different entanglement generation, but not a YES/NO answer!

Regularization

Even for the minimal decoherence prescription, the decoherence is **infinite**.

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Adding a regulator at a length scale σ has 2 effects:

- ▶ It tames decoherence, making it finite
- ▶ It regularizes the pair potential $\propto \frac{1}{r}$ for $r \lesssim \sigma$

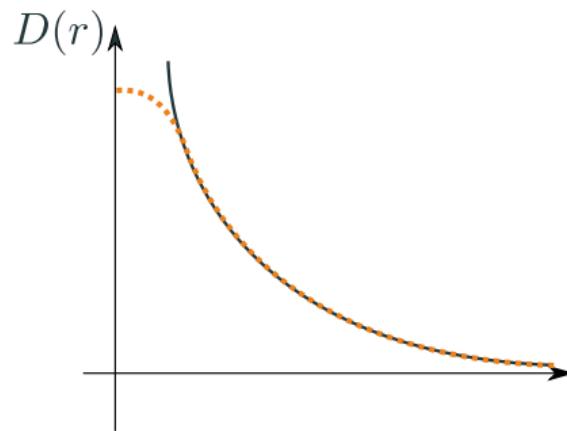
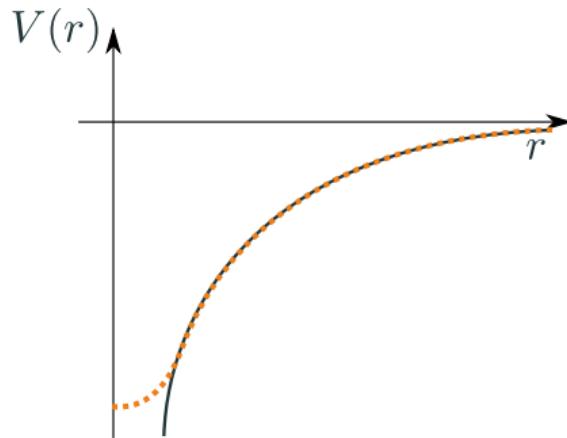
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⇒ there is a **trade-off**.



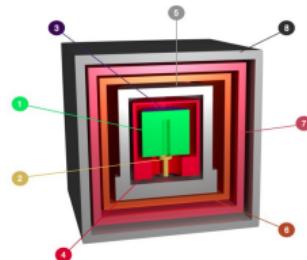
Tests of the cutoff length

Experimentally:

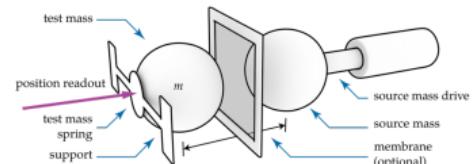
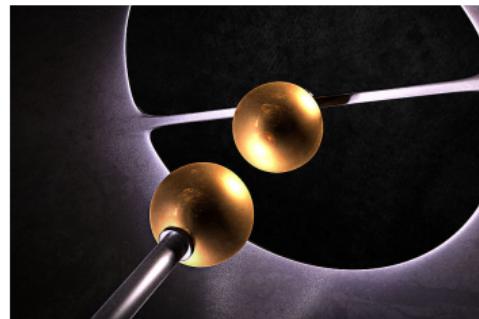
$$10^{-15} \rightarrow 10^{-10} m \ll \sigma \leq 10^{-4} m$$

decoherence constraint gravitational constraint

Importantly $\sigma > \ell_{\text{Compton}} \gg \ell_{\text{Planck}}$.



credit VIP collaboration – Catalina Curceanu



copyright Jonas Schmöle - Aspelmeyer group

Summary

Conceptually: 3 equivalent ways to construct hybrid quantum-classical dynamics

1. **Measurement and feedback** which shows **consistency**
2. **Spontaneous collapse** which shows **empirical effects** + **measurement problem solution**
3. **Quantum Classical PDE** which shows **generality**

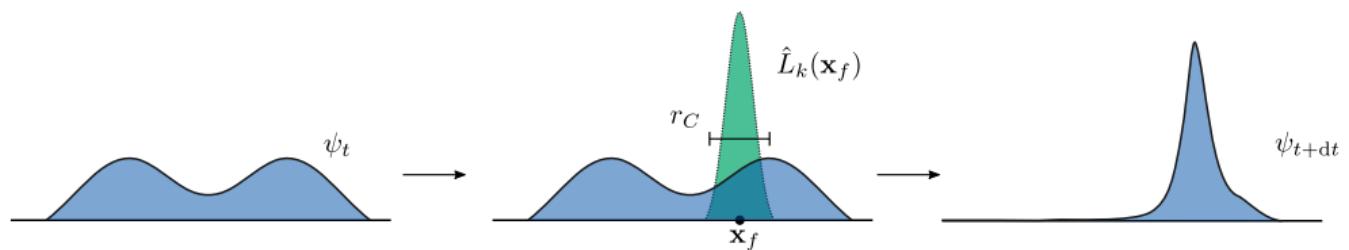
Quantum classical dynamics are *possible, well understood, and classified*

For gravity

- ▶ Newtonian limit: well defined models – minimizing decoherence gives Diosi-Penrose model
- ▶ General case: being explored by Oppenheim et al. – big progress, but not clear all constraints can be met
- ▶ Typically generates spatial entanglement if we ask for low decoherence

Bonuses and Miscellanea

Spontaneous collapse models



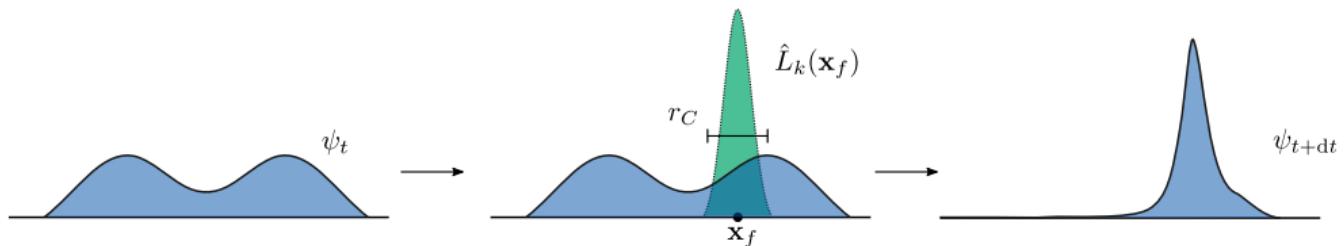
The idea of collapse models

Other names: [Objective / spontaneous / dynamical] [reduction / collapse] [model / program]

Schödinger equation + tiny non-linear bit

$$\frac{d}{dt}\psi_t = -\frac{i}{\hbar}H\psi_t + \varepsilon(\psi),$$

H is the Standard Model Hamiltonian (or non-relativistic approx)



Spontaneous collapse models

Mathematically, (continuous) Markovian spontaneous collapse models are equivalent to continuous measurement of appropriate observables

$$\mathcal{O} \longrightarrow \hat{M}_\sigma(x)$$

Metaphysics – Ontology – beables

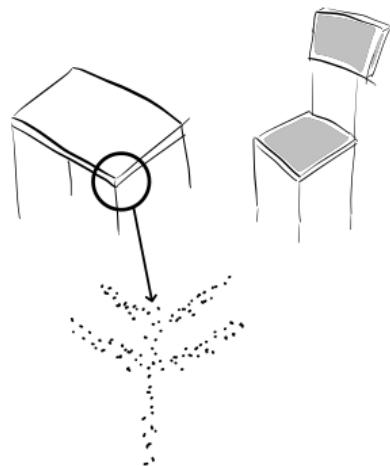
What is real ? What is the world made of ?

1. GRW0 The wave-function ψ_t itself (but infinite literature of subtleties)
2. GRWm The mass density $\langle \hat{M}(x) \rangle$

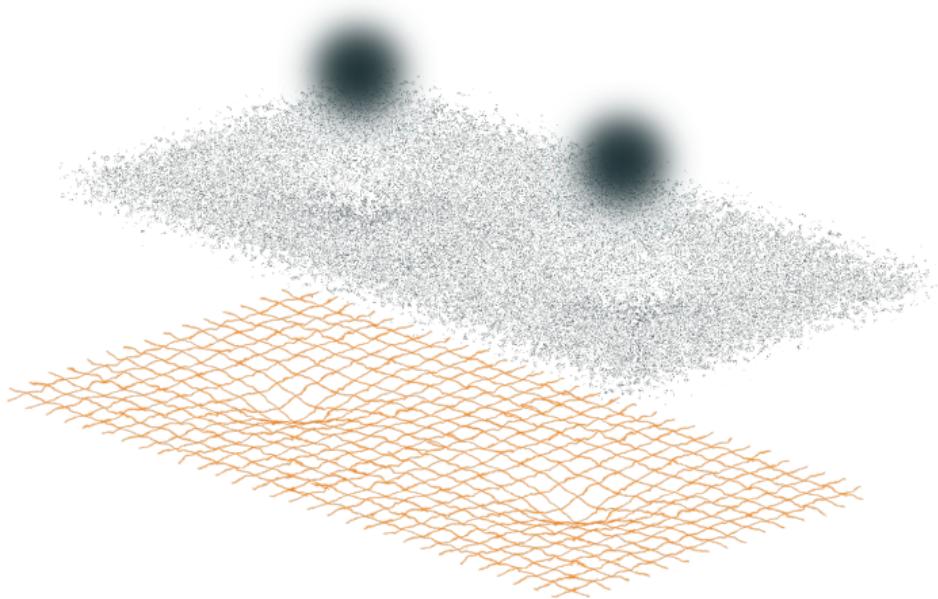
$$\langle \hat{M}(x) \rangle = \sum_k \int dx_1 \cdots dx_n \underset{x \text{ in } k^{\text{th}} \text{ position}}{|\psi(x_1, \dots, x, \dots, x_n)|^2}$$

3. GRWf The events (t_f, x_f) where the wave-function collapse (the flashes) – [Bell's choice!]

Fact: (continuous) flashes = signal



Collapse model picture of hybrid dynamics



“The gravitational interaction is mediated by a stochastic field, which is the **local beable** of the theory”